Week 3 Announcements

- Lecturer – Dr. Ed Michor
  - Weeks 3-4
- Error in HW2

4. Create a MATLAB script that simulates the weight measurement of 10 cans from the normally distributed population described in Problem 3 above. The function `randn` will be useful to do this problem.

http://www-rohan.sdsu.edu/doc/matlab/toolbox/ref/randn.html

A. Have the script report the 10 measurements, and the statistics average and standard deviation. By hand, draw a graphical representation (sketch) of the population and the sample and label them on your sketch.

B. Modify your script so that it repeats part A 5 times and reports the values, the averages, and standard deviations.

C. Of all the 50 measurements
   i. How many are less than the 16 oz advertised? How does that compare with your answer to Problem 2 Part A?
   ii. How many are between 16 and 16.3 oz? How does that compare to the answer in Problem 2 Part B?
Z- Table Refresher

• Please answer the Concept Warehouse question regarding z-tables
Chemical Mechanical Planarization (CMP)

- Process in semiconductor manufacturing
- Uneven layers of material are polished to a desired thickness and uniformity
  - Metals
  - Oxides
- Wafer is held on a “head” and rotated against a polishing “pad” mounted on a platen
- Caustic/Abrasive slurry is flowed between wafer and pad to assist in removal of material
One variable we track is polish time ($x$)
  - Changes for different products
Let’s say that the polish times for **Product Z** are normally distributed
  - $\mu=35s$
  - $\sigma=2.5s$
• We get concerned if the polish time is below 29s
  • Under polishing leads to reworks
  • Metal left on surface can cause shorts between vias
  • Lost time in processing

• How would we calculate the probability that a Product Z wafer polishes less than 29s?
• We get concerned if the polish time is below 29s
  • Under polishing leads to reworks
  • Metal left on surface can cause shorts between vias
  • Lost time in processing

• How would we calculate the probability that a **Product Z** wafer polishes less than 29s?
  • $P(x<29s)$
  • $z=?$
We get concerned if the polish time is below 29s
  - Under polishing leads to reworks
  - Metal left on surface can cause shorts between vias
  - Lost time in processing

How would we calculate the probability that a Product Z wafer polishes less than 29s?
  - \( P(x<29s) \)
  - \( z = \frac{x-\mu}{\sigma} = -2.4 \)
  - But what does \( z \) mean???
• We get concerned if the polish time is below 29s
  • Under polishing leads to reworks
  • Metal left on surface can cause shorts between vias
  • Lost time in processing

• How would we calculate the probability that a Product Z wafer polishes less than 29s?
  • $P(x<29s)$
  • $z = \frac{x-\mu}{\sigma} = -2.4$
  • $z$ tells us that at 29s we’re 2.4$\sigma$ below the mean of 35s
  • The variable $z$ now replaces $x$ in the Standard Normal Distribution
    • $\mu=0$ and $\sigma=1$
  • Math is worked out in $z$-tables
**TABLE 1**

Standard normal curve areas

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The approximate area under the standard normal curve for $z = -2.4$ is $0.0082$.
$z = -2.4$

**TABLE 1**

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\[ z = -2.4 \]

**TABLE 1**

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$z = -2.4$
• We get concerned if the polish time is below 29s
  • Under polishing leads to reworks
  • Metal left on surface can cause shorts between vias
  • Lost time in processing

• For a z of -2.4
  • $p=0.0084$

• This means that the area beneath our curve LESS THAN $z=-2.4$ has a value of 0.0084
• We get concerned if the polish time is below 29s
  • Under polishing leads to reworks
  • Metal left on surface can cause shorts between vias
  • Lost time in processing

• How would we calculate the probability that a Product Z wafer polishes less than 29s?
  • $p(x<29\text{s})=p(z<-2.4)$
  • $p(x<29\text{s})=0.0084$
• We get concerned if the polish time is below 29s
  • Under polishing leads to reworks
  • Metal left on surface can cause shorts between vias
  • Lost time in processing

• How would we calculate the probability that a **Product Z** wafer polishes less than 29s?
  • \( p(x<29s) = p(z<-2.4) \)
  • \( p(x<29s) = 0.0084 \)

• There is a 0.84% chance that we will polish a wafer for less than 29s
  • Happy management?
What factors to consider if 0.84% is reasonable failure rate?
• What factors to consider if 0.84% is reasonable failure rate?
  • Wafers per day (hour/week)

• If we run 2100 wafers per week
  • 300 per day!
  • 2.5 failures per day...
CMP

- What factors to consider if 0.84% is reasonable failure rate?
  - Wafers per day (hour/week)
- If we run 2100 wafers per week
  - 300 per day!
  - 2.5 failures per day...
- Need tighter control!
Suppose we are interested in estimating the mean height of all 2\textsuperscript{nd} year students in college. We take a random sample of 100 and find the sample mean height is $\bar{x} = 67.5$ inches and the sample standard deviation is $S = 4$ inches.
Pearson’s (Fisher’s) ideas

• “What one really gets out of an experiment is a scatter of numbers, not one of which is right but all of which can be used to get a close estimate of the correct value.”

• “The statistical models of distributions enable us to describe the mathematical nature of the randomness”

• “Pearson conceived the measurements themselves rather than the error in the measurements as having the probability distribution”

• “Each of the distributions is identified by four (two) numbers ... the numbers were later called parameters from the Greek for ‘almost measurements’.”
Pearson’s ideas

• “Pearson proposed that these observable phenomena were only random reflections. What was real was the probability distribution. The real ‘things’ of science were not things that we could observe and hold but the mathematical functions that described the randomness of what we could observe. The parameters of a distribution is really what we want to find in a scientific investigation.”

In pairs, identify some ways that we learn engineering and science that discourage us from taking Pearson’s perspective.
Suppose we are interested in estimating the mean height of all 2nd year students in college. We take a random sample of 100 and find the sample mean height is $\bar{x} = 67.5$ inches and the sample standard deviation is $S = 4$ inches.

**What would be our estimate of the population mean height?**

Well instead of giving a single value (67.5 inches), it is more appropriate to give a range, e.g., “it is probably between 66 inches and 69 inches.” Such an estimate is an interval estimate.

What does the size of the interval depend on?

1. The sample size, $n$. The larger the estimate, the greater the precision.
2. The variability in the population ($\sigma$), which can be estimated by the variability in the data ($S$)
3. The level of **confidence** we wish to have.
To see how confidence intervals work, let’s assume that the mean height of 67.5 inches is representative of the population parameter, $\mu$, and the sample standard deviation of 4 inches, is representative of the population parameter, $\sigma$.

**What interval of heights would encompass 95% of the population?**

$P(X > x) = 0.975$.

$P(X < x) = 0.025$.

Statistical Tables are available on the class web site.
To see how confidence intervals work, let’s assume that the mean height of 67.5 inches is representative of the population parameter, μ, and the sample standard deviation of 4 inches, is representative of the population parameter, σ.

What interval of heights would encompass 95% of the population?

\[
P(X > x) = 0.975.\]

Therefore, \( z = 1.96 \quad z = \frac{x - \mu}{\sigma} \)

\[
x = \mu + z\sigma
\]

\[
P(X < x) = 0.025.\]

Therefore, \( z = -1.96 \quad z = \frac{x - \mu}{\sigma} \)

\[
x = \mu + z\sigma
\]

We call this half \( \alpha/2 \)

95% of the population is between 59.7” and 75.3” (write down)
A large college class has 900 students, broken down into section meetings with 30 students each. On the final exam, scores followed a normal distribution with an average of 65 and a standard deviation of 15.

A. If you randomly select one of these students, what is the probability that the selected student scored between 50 and 80 on the final exam?

B. If we consider a section of 30 students as a random sample from this population, will the probability that the average for the entire section is between 50 and 80 be higher or lower or the same as what you calculated in the previous question? Explain. (You do not need to perform any additional calculations.)
Sampling Distribution
Standard Error and Confidence interval

The spread of the *sample* distribution is given by $\sigma_{\bar{x}}$ which is often called the *standard error*. This terminology reflects the fact that we usually do not know the distribution of the *population* so that there is uncertainty in the *sample* data. The variance of sample data is correlated to the degree of uncertainty. It can be shown that the population’s standard deviation, $\sigma$, is related to the standard error by:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \approx \frac{S}{\sqrt{n}}$$

**Confidence Interval.** The interval in which we are reasonably confident a statistical parameter of a *population* lies, based on a finite number of *samples*. Typically we speak of 95% confidence intervals.
Suppose we are interested in estimating the mean height of all 2nd year students in college. We take a random sample of 100 and find the sample mean height is $\bar{x} = 67.5$ inches and the sample standard deviation is $S = 4$ inches.

**What would be our estimate of the population mean height?**

$$P(X > x) = 0.975.$$  
Therefore,  
$$z = 1.96 \quad z = \frac{x - \bar{x}}{S/\sqrt{n}}$$

$$x = \bar{x} + z_{\alpha/2} \frac{S}{\sqrt{n}} = 68.3”$$

$$P(X < x) = 0.025.$$  
Therefore,  
$$z = -1.96 \quad x = 66.7”$$

We are 95% confident that the population mean is between 66.7” and 68.3”
4-4.5 Confidence Interval on the Mean

Figure 4-12  Repeated construction of a confidence interval for $\mu$. 
• The real ‘things’ of science were not things that we could observe and hold but the mathematical functions that described the randomness of what we could observe. The parameters of a distribution is really what we want to find in a scientific investigation.”

Shape
Location (central tendency)
Spread
Studio 3: Bring MATLAB

Kinematic viscosity data (in centistokes [mm$^2$/s]) from a batch chemical process is presented below (read down, then across). The data are from the following journal: *Quality Engineering*, 4, 487-495 (1992), and available on the class drive.

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$\bar{x} = 14.9$

$s = 0.98$
Sampling distribution: Student t

Shape: Normal
Confidence interval of the population mean, \( \mu \)

\[
\bar{x} - t_{\alpha/2, n-1} \frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + t_{\alpha/2, n-1} \frac{s}{\sqrt{n}}
\]

\[
t_{v=n-1} = \frac{x - \mu}{\sigma_{\bar{x}}}
\]

- We are \( 100(1-\alpha) \)% confident that the population mean is between ...
- \( 100(1-\alpha) \)% confidence interval (usually 95%, 99%, 90%)
- Normally distributed population
- Degrees of freedom: \( v = n - 1 \)
4-4.5 Confidence Interval on the Mean

On average, if 20 separate people did experiments, for a 95% CI:

How do these change for 99% CI? 90% CI?

Figure 4.12  Repeated construction of a confidence interval for μ.