HW#2  CE580 Advanced Concrete Design

Consider a concrete girder, unreinforced for shear. The rectangular section has an area A and is subjected to a shear load V. The load induced average shear stress is:

\[ \nu = V / A \]

where V is a normal random variable and A is deterministic. Assume that the stress to produce shear cracking (denoted as \( \nu_c \)) is also a normal random variable. Assume that V and \( \nu_c \) are independent random variables. The statistics of the random variables are as follows:

\( \bar{V} = \) mean shear force = 50 kips
\( \sigma_{\nu} = \) standard deviation of shear force = 15 kips
\( \bar{\nu}_c = \) mean of shear strength = \( 1.39 \sqrt{f_c'} \) for a deterministic concrete compressive strength of 4000 psi.
\( \sigma_{\nu_c} = \) standard deviation of shear strength = \( 0.398 \sqrt{f_c'} \) for a deterministic concrete compressive strength of 4000 psi.

a) If the member is designed such that the central safety factor is 1.2, calculate the required member area.

b) Define a deterministic design load to be:

\[ V_D = \bar{V} + \alpha_{DV} \sigma_{\nu} \]

and find the value of the constant \( \alpha_{DV} \) such that the probability that the load is greater than \( V_D \) is 0.001.

c) If \( V_D \) is written \( V_D = \gamma \bar{V} \), what is the value of constant \( \gamma \) for part (b)? The term \( \gamma \) is called the load factor.

d) Determine a deterministic design cracking stress level to be:

\[ \nu_D = \bar{\nu}_c - \alpha_{DS} \sigma_{\nu_c} \]

and find the value of the constant \( \alpha_{DS} \) such that the probability that the cracking stress is less than \( \nu_D \) is 0.01.

e) If \( \nu_D \) is written \( \nu_D = \phi_S \bar{\nu}_c \), what is the value of constant \( \phi_S \) for part (d)? The term \( \phi_S \) is called the capacity reduction factor.

f) Determine the required member area such that \( \nu_D = V_D / A \).

g) Compute the reliability index (\( \beta \)) and corresponding probability of failure, if failure is defined to be load-induced stress equal or greater than the material cracking stress when the area of the member is determined (1) in part (a) and (2) in part (f).
**Solution**

a) \[ \overline{V} = 1.39 \sqrt{f'_c} = 1.39 \sqrt{4000} = 87.9 \text{ psi} \]
\[ \overline{V} = \overline{50 \text{ Kips}} \]
\[ \overline{D} = \frac{\overline{V}}{A} \]

Set \[ A = \frac{\overline{V}}{\overline{D}} \]

b) \[ V_D = \overline{V} + \alpha_{DV} \overline{V} \]

**What is** \[ P[V > V_D] = 0.001 \]

\[ \frac{1}{1000} \]

How many sigmas?

**Use NORMSINV(0.001)**

\[ = 3.09 \]

\[ V_D = 50 + 3.09(15) = 96.4 \text{ Kips} \]
c) IF \( V_D = \bar{V} \quad \Rightarrow \quad \bar{V} = \bar{V} + \alpha_{DV} \sigma_V \)

\[
\begin{align*}
\bar{V} &= 1 + \frac{\alpha_{DV}}{\bar{V}} \sigma_V \\
Y &= 1 + \alpha_{DV} \text{ COV} \\
Y &= 1 + 3.09(0.3) = 1.93
\end{align*}
\]

\[
\text{COV} = \frac{\sigma_V}{\bar{V}} = \frac{15}{50} = 0.3
\]

\[
\text{WHAT IS } P[ V < V_D ] = 0.01 \quad \Rightarrow \quad \frac{1}{100}
\]

\[
\text{NORMSINV(0.01) = 2.326}
\]

\[
\begin{align*}
\bar{V} &= 87.9 \text{ psi} \\
\sigma_V &= 0.398 \sqrt{4000} = 25.2 \text{ psi} \\
\text{COV}_V &= \frac{25.2}{87.9} = 0.286
\end{align*}
\]

\[
V_D = \bar{V} - \alpha_{DS} \sigma_V = 87.9 - 2.326(25.2) = 29.3 \text{ psi}
\]

\[
\begin{align*}
\bar{V} &= \bar{V} - \alpha_{DS} \sigma_V \\
\sigma_V &= \frac{25.2}{\bar{V}} = 0.286 \\
\text{COV}_V &= \frac{25.2}{87.9} = 0.286
\end{align*}
\]

\[
V_D = \bar{V} - \alpha_{DS} \sigma_V \\
= 87.9 - 2.326(25.2) = 29.3 \text{ psi}
\]
e) \( \phi_s \bar{V}_C = \bar{V}_C = \alpha_{DS} \sigma_{VC} \)

\[ \phi_s = 1 - \alpha_{DS} \cos \theta \]

\[ = 1 - 2.32 \times 0.286 = 0.33 \]

f) \( \phi_s = \frac{\bar{V}_D}{A} \)

\[ A = \frac{V_D}{\sqrt{V_D}} = \frac{1.93}{0.33} \approx 5.83 \]

\[ \frac{96.9 \text{ (1000)}}{29.3} = 3290 \text{ in}^2 \]

4.8 x (a) \# wow

9) \( \beta \)

\[ \text{DEFINITE } F = \frac{V - V}{A} \]

\[ \bar{F} = \bar{V}_C - \frac{V}{A} \]

\[ \sigma_F = \sqrt{\sigma_{VC}^2 + \frac{\sigma_V^2}{A^2}} \]

IF \( F < 0 \) FAILURE
From Prob (a) $A = 682.5 \text{ in}^2$

$$F = 87.9 - \frac{50000}{682.5} = 14.6 \text{ psi}$$

$$\sigma_F = \sqrt{25.2^2 + \left(\frac{15000}{682.5}\right)^2} = 33.4 \text{ psi}$$

$$\beta = \frac{F}{\sigma_F} = \frac{14.6}{33.4} = 0.4378$$

Goal Seek such that

$$\text{NORMSINV} (\beta) = \beta$$

$$P_F = 0.3306$$

33% Big!

From Prob (f) $A = 3290 \text{ in}^2$

$$F = 87.9 - \frac{50000}{3290} = 72.7 \text{ psi}$$

$$\sigma_F = \sqrt{25.2^2 + \left(\frac{15000}{3290}\right)^2} = 25.6 \text{ psi}$$

$$\beta = \frac{F}{\sigma_F} = \frac{72.7}{25.6} = 2.84$$

$$P_F = 0.0022 = 0.2\%$$

By the way, central safety factor for this is $89.9\%$ (8990/3290) = 5.8%