**Shear Cantilever**

\[ V(x) = -\omega(x) \]

**Moment Cantilever**

\[ M(x) = -\frac{\omega x^2}{2} \]

**Shear Main Span**

**Influence Line @ x**

\[ \sum M_{x} = 0 \]

\[ 1(L-x) - R_{a}(L) = 0 \]

\[ R_{a} = \frac{L-x}{L} \]

\[ h_1 + h_2 = 1.0 \]

\[ \frac{h_2}{x} = \frac{h_3}{\alpha L} : \alpha L \left( \frac{x}{L} \right) = h_3 = x \]
\[ A_1 = \frac{1}{2} x (xL) = \frac{1}{2} x^2 L \quad (+) \quad \text{This is a constant.} \]
\[ A_2 = \frac{1}{2} \left( \frac{x}{L} \right) x = \frac{1}{2} x^2 L \quad (-) \]
\[ A_3 = \frac{1}{2} (L-x)(1-x) = \frac{1}{2} \left( \frac{x^2}{L} - 2x + L \right) \quad (+) \]

\[ V(x) = \sum A_i \cdot \omega \]

Consider \[ + \] and \[ - \] Area separately for \[ DL \] is over all areas.

See plot or spreadsheet.

\[ \text{Moment Main Span} \]

\[ A_1 = \frac{1}{2} h_2 xL \]
\[ A_2 = \frac{1}{2} x h_1 + \frac{1}{2} (L-x) h_1 \]

\[ \tan \theta_1 = \theta_1 = \frac{h_1}{x} \]
\[ \tan \theta_2 = \theta_2 = \frac{h_1}{L-x} \]

\[ \theta_1 + \theta_2 = 1 \ \text{rad} \quad \Rightarrow \quad \frac{h_1}{x} + \frac{h_1}{L-x} = 1 \]

\[ h_1 (L-x) + h_1 x = x (L-x) \quad \Rightarrow \quad x h_1 (L-x) + h_1 x = (L-x) h_1 \]
\[ h_1 (L-x) - h_1 x + h_1 x = \frac{Lx - x^2}{2} \quad \Rightarrow \quad h_1 - h_1 x + h_1 x = Lx - x^2 \]
\[ h_1 = x - \frac{x^2}{2} \]

\[ h_2 = x (1-x) \]

\[ \frac{h_2}{\alpha L} = \frac{1}{x} = \left( 1 - \frac{x}{L} \right) \alpha L = h_2 \]

\[ \alpha L - \alpha x = h_2 = \alpha (L-x) \]
\[ M(x) = \sum A_i \cdot w \]

Consider \( + \) - area separately for live load.

For DL must use all areas.

See plot 3: Spreadsheet.

Quick verification:

To find max \( M^+ \) load only between A-B

\[
M_{\text{max}} = (A_1)dx + (A_2)dx = 0
\]

\[
\left( \frac{x^2}{2} - \frac{x^3}{2L} \right)dx + \left( \frac{x}{2} - \frac{x^2}{2L} \right)(L-x)
\]

\[
\left( \frac{Lx}{2} - \frac{x^2}{2} - \frac{4x^2}{2L} + \frac{x^3}{2L} \right)dx
\]

\[
\frac{2x}{L} - \frac{5x^2}{2L} + \frac{L}{2} - \frac{2x}{L} - \frac{8x}{2L} + \frac{3x^2}{2L} = 0
\]

\[
X = \frac{L}{2}
\]

Makes sense:

\[
M_{\text{max}} = \frac{wL^2}{8} \times 784 - k-
\]

Verified in spreadsheet.
SHEAR ENVELOPE
SAP 2000 LOAD COMBINATIONS FOR M/U ENVELOPES

LINE 1

36'

LINE 2

6'

LINE 3

28'

LINE 4

14'

LINE 5

14'

LINE 6

22'

LINE 7

10'

LINE 8

8'

LINE 9

9'

LINE 10

9'

LINE 11

15'

LINE 12

15'

LINE 13

4'

LINE 14

19'

LINE 15
INFLUENCE LINES

THE USE OF INFLUENCE LINES WILL SHOW WHERE TO PLACE THE LL

SHEAR

RIGHT OF B

AT C

BETWEEN B AND C

MOMENT

AT B

DETERMINE B AND C

CHECK ALL