CE 481 HW#4 Problem 1

Estimated Self-weight
\[ w_{\text{selfweightEST}} = 500 \text{ lb/ft} \]

Material Properties
\[ f_y = 60 \text{ ksi} \]
\[ f'c = 5000 \text{ psi} \]
\[ \beta_1 = \begin{cases} 0.85, & \text{if } \left( f'c \geq 4 \text{ ksi}, f'c \leq 8 \text{ ksi} \right) \\ 0.05 \cdot \frac{f'c - 4 \text{ ksi}}{\text{ksi}}, & \text{if } \left( f'c \leq 4 \text{ ksi} \right), 0.85, 0.65 \end{cases} = 0.8 \]

Assume #11 bars, #4 stirrup, and interior exposure
\[ d_b = \frac{11}{8} \text{ in} = 1.375 \text{ in} \]
\[ d_{\text{stirrup}} = \frac{4}{8} \text{ in} = 0.5 \text{ in} \]
\[ \text{cover} = 1.5 \text{ in} \]

Design for Factored Positive Moment Demand first because it is larger
\[ M_u = 951 \text{ kip ft} \]

Pick a reinforcing ratio (could just try 1%)
\[ \rho_{\text{trial}} = \frac{1}{4} \cdot \beta_1 \cdot f'c = 0.017 \]

Determine Nominal Moment Capacity, try reinforcing ratio of 1.7%, \( R = 898 \text{ psi} \) for given \( f'c \) and \( f_y \) from design aid provided in class, want economical design \( \phi = 0.9 \)
\[ R = 898 \text{ psi} \]
\[ \phi_{\text{trial}} = 0.9 \]

Choose \( b \) of 18 in. with \( h = 36 \text{ in.} \) to get 1:2 proportion (Had to increase web width to fit bars in single layer, so design options \( b \) and \( d \) not quite working out)

\[ h = 36 \text{ in} \]
\[ b_w = 18 \text{ in} \]
\[ d = \left( h - \frac{d_b}{2} - d_{\text{stirrup}} - \text{cover} \right) = 33.313 \text{ in} \]
\[ A_{\text{trial}} = \rho_{\text{trial}} \cdot b_w \cdot d = 9.994 \text{ in}^2 \]

Pick #11’s, check if they fit in web width. \( N_b = \# \text{ of } #11 \text{ bars} \)
\[ N_b = 5 \]
\[ s_{\text{horiz}} = b_w - 2 \cdot \text{cover - 2} \cdot d_{\text{stirrup}} - N_b \cdot d_b - (N_b - 1) \cdot d_b = 1.625 \text{ in} \]
\[ A_s = N_b \cdot 1.56 \text{ in}^2 = 7.8 \text{ in}^2 \]

It will fit in web
Assume steel is at yield ($\varepsilon_t > 0.002$)

$$ T := A_s \cdot f_y = 468 \text{ kip} $$

$$ a := \frac{T}{0.85 \cdot f'c \cdot b_w} = 6.12 \text{ in} $$

$$ c := \frac{a}{\beta_1} = 7.65 \text{ in} $$

$$ \varepsilon_t := \frac{\varepsilon_{cu}}{c} \cdot (d - c) = 0.0101 $$

$\phi := 0.9$

Strain in steel $\varepsilon_t > 0.005$ therefore stress in steel is at yield and $\phi = 0.9$ economical

$$ M_n := T \cdot \left( d - \frac{a}{2} \right) = 1180 \text{ kip} \cdot \text{ft} $$

Determine Design Strength

$$ \phi M_n := \phi \cdot M_n = 1062 \text{ kip} \cdot \text{ft} \quad > \text{Mu OK! This is pretty close!} $$

Check Self-weight

$$ w_{selfweight} := b_w \cdot h \cdot 150 \frac{\text{lb}}{\text{ft}^3} = 675 \frac{\text{lb}}{\text{ft}} $$

Check As min

$$ A_{smin} := \frac{\left(200 \text{ psi} \cdot b_w \cdot d\right)}{f_y} = 2 \text{ in}^2 $$

$$ A_{smin} := \frac{\left(3 \sqrt{f'c \text{ psi} \cdot b_w \cdot d}\right)}{f_y} = 2.12 \text{ in}^2 \quad \text{This controls} $$

As > Asmin ok

Design for Factored Negative Moment demand given the cross section above. Due to smaller moment (570 kip-ft), the number of bars will be proportionally smaller or about 60% as much as for M+. ~4.6 in^2. Use #9s. Added notions with n to designate negative moment design.

$$ N_{bn} := 4 \quad A_{sn} := N_{bn} \cdot 1.0 \text{ in}^2 = 4 \text{ in}^2 \quad d_{bn} := \frac{9}{8} \text{ in} = 1.125 \text{ in} $$

$$ dn := h - \frac{d_{bn}}{2} - d_{stirrup} - \text{cover} = 33.438 \text{ in} $$

Assume negative steel is at yield ($\varepsilon_t > 0.002$)

$$ Tn := A_{sn} \cdot f_y = 240 \text{ kip} $$

$$ an := \frac{Tn}{0.85 \cdot f'c \cdot b_w} = 3.14 \text{ in} $$

$$ cn := \frac{an}{\beta_1} = 3.92 \text{ in} $$

$$ \varepsilon_{tn} := \frac{\varepsilon_{cu}}{cn} \cdot (dn - cn) = 0.0226 $$

$\phi := 0.9$
Strain in steel $\varepsilon_{tn} > 0.005$ therefore stress in steel is at yield and $\phi = 0.9$ economical

$$M_n := T_n \cdot \left( d_n - \frac{a_n}{2} \right) = 637 \text{ kip \cdot ft}$$

Determine Design Strength

$$\phi M_n := \phi \cdot M_n = 574 \text{ kip \cdot ft}$$

As > As min ok same as before

Design Sketch
CE 481 HW#4 Problem 2

Span Length $L := 22 \text{ ft}$

Material Properties

\[ f_y = 60 \text{ ksi} \quad f'_c = 4500 \text{ psi} \]

\[ \beta_1 = \begin{cases} \frac{f'_c - 4 \text{ ksi}}{ksi}, & \text{if } \left( f'_c \leq 4 \text{ ksi} \right) \text{, } 0.85 - 0.05 \cdot \frac{f'_c - 4 \text{ ksi}}{ksi}, & \text{if } \left( f'_c \leq 8 \text{ ksi} \right) \text{, } 0.85, 0.65 \end{cases} = 0.825 \]

Prescribed Geometry

\[ h := 26 \text{ in} \quad b_w := 14 \text{ in} \]

Given #4 stirrup, and interior exposure

\[ d_{stirrup} := \frac{4}{8} \text{ in} = 0.5 \text{ in} \quad \text{cover} := 1.5 \text{ in} \]

Compute Self-weight

\[ w_{selfweight} := b_w \cdot h \cdot 150 \frac{lb}{ft^3} = 379 \frac{lb}{ft} \]

Compute $M_u$, given large LL compared to DL, $1.2D + 1.6L$ will control. Also simple support means that to get worst moment requires loading entire span. If whole span is loaded easiest way to compute $M_u$ is to calculate $w_u$.

\[ w_d := 1100 \frac{lb}{ft} \quad w_l := 2300 \frac{lb}{ft} \]

\[ w_u := 1.2 \cdot \left( w_{selfweight} \right) + 1.2 \cdot \left( w_d \right) + 1.6 \cdot \left( w_l \right) = \left( 5 \cdot 10^3 \right) \frac{lb}{ft} \]

\[ M_u := \frac{w_u \cdot L^2}{8} = 330028 \text{ lb} \cdot \text{ft} \]

\[ M_u := 201 \text{ kip} \cdot \text{ft} \]

Pick a reinforcing ratio (just try 1%) and try #9 bars

\[ \rho_{trial} := 0.01 \quad d_b := \frac{9}{8} \text{ in} = 1.125 \text{ in} \]

\[ d := \left( h - \frac{d_b}{2} - d_{stirrup} - \text{cover} \right) = 23.438 \text{ in} \]

\[ A_{strial} := \rho_{trial} \cdot b_w \cdot d = 3.281 \text{ in}^2 \]

\[ A_s := A_{strial} \cdot 1.0 \text{ in}^2 = 2 \text{ in}^2 \]

Pick #9 bars $N_b = \# \text{ of } #9 \text{ bars}$

\[ N_b := 2 \quad A_s := N_b \cdot 1.0 \text{ in}^2 = 2 \text{ in}^2 \]
Check if they fit in web width.

\[
b_{\text{w,required}} := 2 \cdot \text{cover} + 2 \cdot d_{\text{stirrup}} + N_b \cdot d_b + (N_b - 1) \cdot 1 \text{ in} = 7.25 \text{ in}
\]

**It will fit in web**

Assume steel is at yield (\( \varepsilon_t > 0.002 \))

\[
\varepsilon_t := \frac{\varepsilon_{cu}}{c} = 0.003 \quad \varepsilon_{cu} := 0.003
\]

\[
T := A_s \cdot f_y = 120 \text{ kip} \quad a := \frac{T}{0.85 \cdot f'_c \cdot b_w} = 2.24 \text{ in} \quad c := \frac{a}{\beta_1} = 2.72 \text{ in}
\]

\[
\varepsilon_t := \frac{\varepsilon_{cu}}{c} \cdot (d - c) = 0.0229 \quad \phi := 0.9
\]

Strain in steel \( \varepsilon_t > 0.005 \) therefore stress in steel is at yield and \( \phi = 0.9 \) economical

\[
M_n := T \left( d - \frac{a}{2} \right) = 223 \text{ kip \cdot ft}
\]

Determine Design Strength

\[
\phi M_n := \phi \cdot M_n = 201 \text{ kip \cdot ft} \quad > \text{Mu OK! This is pretty close!}
\]

Could go to 2 #10’s but ok.

Check As min

\[
A_{\text{min}} := \frac{(200 \text{ psi} \cdot b_w \cdot d)}{f_y} = 1.09 \text{ in}^2
\]

\[
A_{\text{min}} := \frac{(3 \sqrt{f'_c \text{ psi} \cdot b_w \cdot d})}{f_y} = 1.1 \text{ in}^2 \quad \text{This controls}
\]

**As > Asmin ok**

Design Sketch