Controlled Passive Dynamic Running Experiments With the ARL-Monopod II

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Abstract—This paper presents the expansion and implementation of the controlled passive dynamic running (CPDR) strategy for legged robots, previously presented by the authors. The CPDR exploits the underlying passive dynamic operation of the robot’s mechanical systems to reduce the energy spent for locomotion. Meanwhile, it ensures the stability of the vertical and forward motions as the robot speed varies. An “adaptive energy controller” stabilizes the hopping height accurately over a range of operating conditions. The passive dynamic derivations for the Monopod, together with the foot-placement algorithm and model-based joint controllers, are used to control the forward speed about the passive operation trajectories. New locomotion variables are used for robust synchronization between the hip-body and the leg oscillations. ARL-Monopod II achieved a speed of 1.25 m/s with specific resistance (a measure for energy cost of locomotion) of 30% of the earlier robot ARL-Monopod I, its predecessor, due to the newer hip and leg design and application of the CPDR control strategy.

Index Terms—Control, legged robot, monopod, passive dynamics.

I. INTRODUCTION

AUTONOMOUS robots need to operate for prolonged periods of time with their limited on-board energy. Achieving the required efficiencies can be particularly challenging for legged robots, which spend a great deal of energy on periodic acceleration and deceleration of the legs and nonrecovered negative work. The energy required for periodic motions, like the swinging of legs, can be reduced by incorporating compliant elements in the joints to allow the limb motions to be generated via their “passive dynamic” behavior. This principle is already at work in nature. For example, the metabolic cost of running is reduced through elastic properties of the muscles, tendons, and bones in mammalian running [2]–[7]. However, passive motions are not limited to mass-spring systems. In low-speed human walking, each leg and the body constitute an inverted pendulum whose unstable, unforced behavior results in a considerably efficient gait of motion [8]. Walking, running, writing, or swimming, among many other physical activities, are easier tasks to perform if done at certain frequencies or speeds that match natural limb motions.

The idea of a passive hopper first appeared in [9], suggesting a hopping lunar vehicle to reduce energy requirements. The application of passive dynamics to dynamically stable robots was sparked by the work of Matsuoka [10] and numerous innovative hopping robots developed by Raibert [11]. Most hopping robots use a pogo-stick mechanism with a thrust actuator to stabilize their vertical oscillations. Many other dynamically stable running hoppers are developed [11]–[15] based on compliant knee joints. An example of a self-stabilizing passive walking machine was McGeer’s Biped [16]—a two-legged walking robot stepping downwards on a slope, propelled purely by gravity. Numerous researchers have investigated passive bipedal locomotion and their stability via simulations [17]–[19]. Self-stabilizing quadrupeds were studied in [20], and an intelligent control approach was proposed for a similar system in [21].

More recently, a number of biologically inspired multilegged robots have been realized. With these robots, a new field emerged that brings roboticists and biologists together who aim at understanding the fundamental principles of animal locomotion and applying them to robots. Among the latest machines are the simple mechanical robotic designs with four legs [22], [23] or six legs [24]–[26]. Increasingly, emphasis is placed on finding new ways to incorporate and exploit elastic elements in the limbs [27].

For controlled passive dynamic running (CPDR) implementation, we design the robot in such a way that its natural behavior is close to running itself. That requires an additional compliance at the hip joint and proper initial conditions. The importance of CPDR is best understood by comparing the energetics of ARL-Monopod II with those of ARL-Monopod I without the passive hip and CPDR implemented. The experiments reported in [13] showed that the ARL-Monopod I (without hip compliance) expended 40% of its total energy requirement, at a speed of 1.2 m/s, to swing the leg. This is the energy spent on accelerating and decelerating the leg about the hip joint to attain the proper angle before each touchdown. Much of this energy can be saved using a spring connecting the leg and the body at the hip joint. In [28], it is shown that with the right parameters and initial conditions, a robot model similar to our Monopod with hip and leg springs can run for a number of steps before falling. In previous research [1], the basics of CPDR with a stabilizing controller was introduced using simulation and a simplified one-legged robot model. ARL-Monopod II was later developed to validate this strategy. Preliminary results of ARL-Monopod II running have been reported in [29], but more details are available in [30].
In this paper, we present the robot design, the modifications to the strategy for the experimental system, the new controllers, and the energetics comparisons with ARL-Monopod I.

This paper is organized as follows. The design of ARL-Monopod II, the experimental setup, and differences with Monopod I are highlighted in Section II. The basics of the CPDR method is briefly discussed in Section III. The controller derivations to achieve stable and accurate control of vertical height and forward speed and related experiments are included in Section IV. Energetics of the ARL-Monopod II are later analyzed for different speeds and compared against Monopod I in Section V. Sections VI and VII discuss the future work and conclude the results.

II. ARL-MONOPOD II

The ARL-Monopod II (Fig. 1) main body is constrained to move in a vertical plane via a planarizer, which consists of two linear and one rotary stages connected at the hip. The robot is about 0.7-m tall, weighs approximately 18 kg, and is self-contained, i.e., the data-acquisition system, control computers, and power supply are all embedded. The robot runs on a strong treadmill whose velocity can be commanded by the robot automatically, or manually by the operator. The treadmill actuator is designed to be strong enough to keep its velocity disturbance small under strong robot foot impacts, particularly at high speeds with large leg angles. The robot should maintain a vertical hopping height and simultaneously maintain its horizontal position at the center of the treadmill at any forward speed. The transient dynamics of the robot is slightly different when it runs on a treadmill compared with running on the ground, but in steady state, the behaviors are similar [31], [32].

Fig. 2 shows the kinematic arrangement of the robot with important variables shown. The robot’s body and the leg are attached approximately at their centers of mass via the revolute hip joint. The planarizer accommodates the body with three degrees of freedom (DOFs): the horizontal, vertical, and pitching motions $x$, $z$, and $\phi$, all measured at the hip joint. The leg introduces two more DOFs: the hip rotation $\gamma$ and the leg extension (knee joint) $r$. These five DOFs can be considered as “motion DOFs.” The hip and leg actuators feature precision ball screws with springs connected in series. The compliant elements in series with actuators give rise to two new “actuation DOFs” $\tau_h$ at the hip and $\tau_l$ at the knee. Therefore, the total number of DOFs is seven with only two control inputs—the hip and the leg motor torques $\tau_h$ and $\tau_l$. In Fig. 2, $\theta = \gamma + \phi$ is the absolute leg angle with respect to the vertical, and $X_{fl}$ represents the foot position with respect to the hip measured horizontally.

The leg actuator incorporates a helical spring installed between the lower leg and the ball-nut assembly. Lift-off and touchdown happen at a fixed spring length, but not at fixed body height, as it depends on the actuator position. The spring at its maximum length is almost at rest to keep the impact loads small at lift-off. The hip joint incorporates a double pulley-spring system with four surgical latex tubing acting as spring. The range of required stiffness and displacement for this robot demands a high-strain-and-energy-density material. Latex offers a high-strain extension of 700% of the initial length. The rubber’s energy density per weight is higher than steel, mainly due to its higher strain. Through experiments detailed in [30], we found three regions of behavior: nonlinear softening behavior for small deflections; linear behavior for a wide range of deflections; and nonlinear stiffening behavior for large deflections. These springs cannot stand compression, so they are preloaded to operate within their large linear range.

The robot parameters are listed in Table I. Throughout this paper, we use a particular nomenclature for variable referencing, such as subscripts and superscripts, as presented in Table II. ARL-Monopod I was not suitable for CPDR implementation since its hip joint was directly actuated using inextensible Spectra Fibres. The leg design with larger impact losses was not as elegant as the improved Monopod II leg that releases almost all the potential energy stored in the leg spring before lift-off. The leg design improvement is partly responsible for reduced power requirements of ARL-Monopod II. Further details of the two leg and hip designs are available in [30].
TABLE I
IMPORTANT PARAMETERS OF ARL-MONOPOD II

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
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<tbody>
<tr>
<td>$J_b$</td>
<td>body inertia</td>
<td>0.75 kgm²</td>
</tr>
<tr>
<td>$J_l$</td>
<td>leg inertia</td>
<td>0.12 – 0.19 kgm²</td>
</tr>
<tr>
<td>$k_h$</td>
<td>hip spring stiffness (linear)</td>
<td>3,800 N/m</td>
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<tr>
<td>$k_l$</td>
<td>leg spring stiffness</td>
<td>6,312 N/m</td>
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<tr>
<td>$m_b$</td>
<td>body mass</td>
<td>11.0 kg</td>
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<tr>
<td>$m_l$</td>
<td>leg mass</td>
<td>4.6 kg</td>
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<tr>
<td>$r_l$</td>
<td>leg ball screw lead</td>
<td>0.796 mm/rad</td>
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<tr>
<td>$r_h$</td>
<td>hip ball screw lead</td>
<td>2.5 mm/rad</td>
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<tr>
<td>$m_u$</td>
<td>upper leg mass</td>
<td>14.98 kg</td>
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<td>$m_{ul}$</td>
<td>lower leg mass</td>
<td>0.372 kg</td>
</tr>
<tr>
<td>$m_{nat}$</td>
<td>nut mass</td>
<td>0.4 kg</td>
</tr>
<tr>
<td>$z_{cg,lt}$</td>
<td>lower leg C.G. measured from toe</td>
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<td>$z_{cg,ut}$</td>
<td>upper leg C.G. measured from hip</td>
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<td>$z_{cg,b}$</td>
<td>body C.G. measured from hip</td>
<td>-0.059 m</td>
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<tr>
<td>$p_0$</td>
<td>home position of the leg actuator</td>
<td>0.106 m</td>
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<tr>
<td>$r_0$</td>
<td>leg length (when $p_l = 0$ in flight)</td>
<td>0.64 m</td>
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<tr>
<td>$b_0$</td>
<td>leg spring pre-compression</td>
<td>0.0064 m</td>
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TABLE II
DESCRIPTION OF INDICES

<table>
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<tr>
<th>Symbol</th>
<th>Value</th>
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<tbody>
<tr>
<td>$d$</td>
<td>desired value</td>
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<tr>
<td>$l$</td>
<td>lower leg</td>
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<tr>
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<td>upper leg</td>
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<td>$l$</td>
<td>lift-off</td>
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<tr>
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</tr>
<tr>
<td>$a$</td>
<td>actuator</td>
<td>body:upperleg</td>
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</table>

For proper timing, the robot’s step period $T_{step}$ should match the hip natural oscillations period, which is determined by hip stiffness and body and leg inertia. The relative motion of the foot with respect to the hip can be approximated as a sinusoid. The natural frequency of the hip oscillations being fixed, the amplitude is left as the only means of adapting to various forward speeds: a faster motion requires a larger stride and larger hip oscillations.

For vertical oscillations, the overall period $T_{step}$ is the sum of the flight period $T_f$ and the stance period $T_s$. Contrary to the hip motion, the vertical period can change, via $T_f$ if the hopping height changes, and via $T_s$ due to the “leg spring stiffening” effect at higher speeds [1], [33]. The duty factor, defined by the ratio $\rho = T_s/T_{step}$ once chosen, determines the leg spring stiffness. High leg stiffness is normally desired, because it shortens the stance period and reduces the duty factor that helps the assumption of sinusoidal foot motion during stance, as shown in Fig. 4. For the real system, the stiffness cannot be too large, otherwise the stance time is too short for the controller to add enough energy to the system to maintain the hopping motion.

For passive operations design, the robot physical parameters $m_b, m_l, J_b, J_l, k_h, k_l$ can be used to determine the hip oscillations frequency $\omega_h = 2\pi / T_{step} = \sqrt{(k_h/J_b)}$, the leg oscillations frequency $\omega_l = \pi \sqrt{m_l/k_l}$, and the duty factor $\rho = T_s/T_{step} = \omega_h/2\omega_l$, where $m$ is the total robot mass and $J_r = (J_b J_h)/(J_l + J_h)$ the equivalent hip inertia. Assuming the robot is released at apex with leg and body angles set to zero at time zero, the foot position relative to the hip is described by $x_f = -r_0 \sin \theta^*(t) = -r_0 \sin (\theta^* \sin \omega_h t)$. Knowing the duty factor, the average velocity $\bar{x}$ during stance can be calculated by dividing the total foot displacement by $T_s$ to obtain the key relationship between a given robot velocity and the hip oscillations amplitude $\dot{\theta}^*$ required for passive operation

$$\bar{x} = -\frac{2r_0}{T_s} \sin \left[ \frac{\dot{\theta}^* \sin ((1 - \rho)\pi)}{\sin ((1 - \rho)\pi)} \right]$$

$$\dot{\theta}^* = \frac{\arcsin (T_s/\bar{x}/2r_0)}{\sin ((1 - \rho)\pi)}.$$ (1)

The body oscillations amplitude is also found via $\dot{\phi}^* = -(J_l/J_r)\dot{\theta}^*$. Starting with touchdown angle and flight time, the desired apex height $z_{apex}$ is calculated via

$$z_{ap} = \frac{g}{8} (1 - \rho)\dot{\phi}^2 T_{step}^2 + r_0 \cos \left[ \frac{\dot{\theta}^* \sin ((1 - \rho)\pi)}{\sin ((1 - \rho)\pi)} \right]$$ (2)

where $r_0$ is the leg length at touchdown. The reader is strongly recommended to refer to [1] for a detailed version of the aforementioned. So far, the conditions resulting in passive dynamic

### Fig. 3
Locomotion phases during one cycle.

### Fig. 4
Foot position with respect to hip with linear approximations of stance phase.

### III. PASSIVE DYNAMIC RUNNING

For ARL-Monopod II, running can be approximated by combination of two synchronized oscillatory motions: the vertical hopping motion of the body mass over the leg spring; and the rotational counter-oscillations of the leg and body connected via the hip spring. A locomotion cycle is illustrated in Fig. 3 to help visualize how the two oscillations make a complete running step. A cycle consists of the flight and stance phases. The flight starts with lift-off when the toe leaves the ground, and ends with touchdown when the toe touches again. The stance phase starts with touchdown and ends with lift-off. The apex and bottom are events that occur at maximum and minimum body heights, respectively. With proper selection of initial conditions and in the absence of actuator motion, the robot takes two steps and falls rapidly. This running, though unstable, is entirely based on the robot’s passive dynamic behavior. The following derivations are highlights of the CPDR calculations. For more details, refer to [1].
operation and the form of the motions are introduced. The next question is the control and stability of ARL-Monopod II ensuring its operation around the motions obtained earlier.

IV. CONTROL

The overall objective of the controller is to use the passive dynamic motion as reference trajectories, track them in a stable and robust fashion, meanwhile adapting its motions to varying forward velocity. There are two main control tasks: the hopping height control and the planar motion control, including the leg angle, hip angle, and forward velocity. The leg actuator controls the hopping height during stance and the leg length during flight. The hip actuator can control the leg angle during flight and the body attitude during stance.

To be implemented on ARL-Monopod II, the CPDR strategy faces a number of new challenges compared with the original CPDR strategy tested with simulation [1]. The actuators dynamics are not negligible here and affect the control derivations, hence will not be treated as position input devices. The new leg design also allows touchdown and lift-off to happen at various heights, which was not the case for ARL-Monopod I and the original CPDR work. The controllers presented in this paper are adapted to resolve these additional complexities faced by Monopod II, and they perform well, even in the face of parameter uncertainties.

A. Hopping Height Control

The performance of the hopping height control is of paramount importance to the success of the rest of the controllers which run in harmony with the vertical motion. The objective is to achieve a desired “apex height” during flight with the leg actuator torque available as input during stance. The challenge is to relate the apex height error of the last step to a control action for the upcoming stance phase so that the error decreases for the next apex. This is achieved via the adaptive/learning energy control method as follows.

1) Energy as a Control Variable: The total energy of a system is the sum of the all potential and kinetic energies. However, we define the “vertical energy” as the total energy of the system only calculated based on the vertical components of the motion by

\[ E_v = E_{per} + E_{kin,v} \]

where \( E_{per} \) includes the gravitational and elastic potential energies and the kinetic energy \( E_{kin,v} \) is defined to be based on only the robot’s vertical velocity components. Assuming the leg actuator settles to its desired value before apex, and knowing that vertical speed is zero at apex, the only term remaining in the potential energy at apex is the apex height. This provides a simple one-to-one correspondence between the apex energy \( E_{a} \) and the apex height \( z_{a} \)

\[ E_v = M g z_{a} + E_{t0} \]  

where \( E_{t0} \) is a constant determined by the energy reference and \( M \) is the total robot mass. The objective of the controller can now change from maintaining the desired height \( z_{a,d} \) to regulating \( E_v \) to its desired value \( E_{a,d} \) during stance, as related by (3). This relationship is important since it does not depend on lift-off and touchdown conditions.

2) Energy Feedback Controller: A stable feedback controller is required to regulate the energy at its desired value. We define the step \( n \) to start at the \( n \)th apex and to end at the \((n+1)\)th apex. The vertical energy of the robot at the \( n \)th apex is denoted by \( E_{a,n} \). The energy at the next step \( E_{a,n+1} \) is a function of the current apex energy \( E_{a,n} \), the added energy by the actuator during the \( n \)th step \( E_{act,n} \), and the energy loss during the step \( E_{loss,n} \) described by

\[ E_{a,n+1} = E_{a,n} + E_{act,n} - E_{loss,n} \]  

We realized from the experiments that the value of the energy loss \( E_{loss} \) (about 4 J per step) is not negligible and not even known until the step ends—too late to be compensated. One possibility is to use a loss estimate \( \hat{E}_{loss} \) and correct the estimator based on observed values at the end of each step. The actual loss during the past step is known as

\[ E_{loss,n} = E_{a,n} - E_{a,n+1} - E_{act,n} \]  

which can be used as a simple estimate \( \hat{E}_{loss} \). The estimated loss can be used with (4) to compute the reference value for the leg actuator energy controller as

\[ E_{act,d} = E_{a,d} - E_{a,n} + \hat{E}_{loss} \]  

Several loss-estimation alternatives were experimented on, such as a lookup table or moving average, but the following model-based adaptive estimator:

\[ \hat{E}_{loss} = K_{loss} (E_{a,n} - E_0) \]

showed superior performance. \( K_{loss} \) is the only parameter in the above and \( E_0 \) is a known constant that depends only on the potential energy of the system at lift-off with respect to a known reference. It can be verified that most of the possible losses throughout a step are proportional to the square of the lift-off velocity or proportional to \( (E_{a,n} - E_0) \). Note that losses are mostly due to impacts and dry friction for a hopping machine. A simple update formula for \( K_{loss} \) can be

\[ K_{loss,n+1} = K_{loss,n} + \frac{E_{act,n} - E_{loss,n+1}}{E_{a,n} - E_0} \]  

If \( K_{loss,n} \) was known before the end of the step \( n \) and used instead of \( K_{loss,n} \) for the \( n \)th step, then it would have resulted in an accurate estimate for the loss of \( E_{loss,n} = E_{loss,n} \) to be used for control. To try that, \( E_{loss,n} \) in (4) is replaced by (6) to get

\[ E_{a,n+1} = E_{a,n} + \hat{E}_{loss} - E_{loss,n} \]  

and to find the actual loss as

\[ E_{loss,n} = \hat{E}_{loss} + E_{a,n} - E_{a,n+1} \]
If we start with $\dot{E}^n_{\text{loss}} = E^n_{\text{loss}}$ and use (7) and (10), then

$$K^\gamma_{\text{loss}} (E^\gamma_{\text{apex}} - E_0) = E^\gamma_{\text{loss}}$$

$$= E^\gamma_{\text{loss}} + E_{\text{apex,d}} - E^\gamma_{\text{apex}}$$

$$= K^\gamma_{\text{loss}} (E^\gamma_{\text{apex}} - E_0) + E_{\text{apex,d}} - E^\gamma_{\text{apex}}$$

$$K^\gamma_{\text{loss}} = K^\gamma_{\text{loss}} + \frac{1}{2} \left( E_{\text{apex,d}} - E^\gamma_{\text{apex}} \right) / (E^\gamma_{\text{apex}} - E_0).$$

(12)

Dividing both sides of (11) by $(E^\gamma_{\text{apex}} - E_0)$ leads to the parameter update of (8). Implementation of this estimator caused small steady-state oscillations in height control. To remedy, the average of the last two estimated parameters was used with the following form:

$$K^\gamma_{\text{loss}} = K^\gamma_{\text{loss}} + \frac{1}{2} \dot{E}_{\text{apex,d}} - E^\gamma_{\text{apex}} / (E^\gamma_{\text{apex}} - E_0).$$

(12)

The new update formula is less “forgetting” than (8) and eliminates the steady-state oscillation.

3) Stance Energy Controller: The last stage of the energy controller is to determine how to add $E_{\text{act,d}}$ and continually measure the added energy $E_{\text{act}}(t)$. The energy input to the system is measured via direct integration of the actuator output power determined by the actuator force $F_l$ and the velocity $\dot{h}$

$$E_{\text{act}}(t) = \int_{t_{\text{vd}}}^{t} F_l \ddot{h} \, dt$$

(13)

where $t_{\text{vd}}$ is the touchdown time. The total vertical added energy of the actuator during stance can be found by setting the integral limits to $t_{\text{vd}}$ and $h_{\text{vd}}$. Assuming a linear leg spring with constant $k_l$ and small damping forces, the spring force in (13) can be related to deflection via $F_l = k_l (q_0 - q_t + \dot{q})$, which simplifies the measurements. The added vertical energy will be slightly different for when the robot moves forward and the leg has an angle with respect to vertical. In that case, both the actuator velocity and the force are projected to the vertical direction (multiplied by the cosine of the leg angle), to evaluate (13). The energy error can be related to the actuator desired velocity $\dot{q}_{\text{vd}}$ by

$$\dot{q}_{\text{vd}} = K_{E1} \left( E^\gamma_{\text{act,d}} - E_{\text{act}}(t) \right)$$

with the gain $K_{E1}$. This simple control can add or remove energy from the system as required. The commanded velocity is zero when the actual and desired energy are equal. The desired actuator velocity $\dot{q}_{\text{vd}}$ is still to be controlled. A simple velocity controller with feed-forward compensation of $F_l$ is used

$$\tau_l = -K_{E2} (\dot{q}_{\text{vd}} - \dot{h}) + \tau_l F_l$$

(14)

where $\tau_l$ is the leg ball screw lead in m/rad. The gain $K_{E2}$ is to be selected in a way that the actuator displacement does not exceed its limit before lift-off.

There are practical limitations on the rate of energy injected. The energy error is larger at touchdown and requires large actuator velocities. However, in this condition, the leg force is smaller, which reduces the rate of energy input. Increasing the gain $K_{E1}$ increases the speed and damping forces and can saturate the actuator. This issue is resolved by introducing a velocity correction factor, which scales down the desired actuator speed when the leg force is small. The correction factor $\zeta$ is defined by

$$\dot{q}_{\text{vd}} = K_{E1} \zeta \left( E^\gamma_{\text{act,d}} - E_{\text{act}} \right)$$

(15)

where

$$\zeta = 1 - q_{\text{lvd}} / q_{\text{l0}}$$

(16)

and $q_{\text{lvd}} = q(t_{\text{vd}})$. The factor $\zeta$ attains its maximum at maximum spring compression and vanishes at touchdown and lift-off. Most of the energy transfer is done around the bottom point. A block diagram of the energy controller is shown in Fig. 5. During flight, a simple proportional-derivative (PD) controller with feed-forward compensation of gravitational loads forces the actuator back to the home position.

4) Vertical Hopping Experiments: The performance of the controllers, when implemented on the robot depends heavily on how fast the loss estimator converges. Experiments showed fast convergence to the actual value within just a few steps, as illustrated in Fig. 6. At the beginning of the run, the robot starts at the bottom, resting on its leg. By a swift actuator motion against the spring to push the body upward, it starts the first flight phase.

Fig. 7(a) and (b) depict the body height and actuator position correspondingly. Over a large range of experiments, the robot stabilized to desired hopping heights within the first few hops. Fig. 8 shows that energy controller almost reaches its desired
value. As seen in Fig. 8, the actuator is not physically able to deliver the large required energy within one or two steps, but catches up quickly. A small steady-state energy error is observed in the experiments. This error can also vary with the robot’s forward speed, affecting the performance of the vertical controller. The robot can learn from its past step to correct its controller similar to the adaptive loss estimator presented before. To eliminate the steady-state error, the energy offset $E_{\text{act,}d} - E_{\text{act}}(t)$ can be measured at the end of each stance and be compensated in the next step. If the ratio of the error to the desired input energy is defined by $\psi_E = (E_{\text{act,}d} - E_{\text{act}}(t))/E_{\text{act,}d}$, the energy controller of (15) can be modified to cancel the offset as

$$\dot{\mu}_d(t) = K_{E1} \zeta \left( (1 + \psi_E^2) E_{\text{act,}d}^\infty - E_{\text{act}}(t) \right). \quad (17)$$

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Fig. 6. Estimated energy loss (solid line with circles at apex) converges to the measured loss (dashed line with stars at the apex).

Fig. 7. (a) Body height response to a fixed desired height starting from rest. (b) The actuator motion in response to the energy control of stance and position control of flight. Each “+” sign on the graphs shows a lift-off and each “o” shows a touchdown.

Fig. 8. Solid curves show the measured input energy, and the dashed curves show the desired level. On the curves, “+” is for lift-off, “o” for touchdown, and “s” is for apex.
The factor \( \psi_E^{(n)} \) follows a similar update formula as the loss estimator

\[
\psi_E^{(n)} = \psi_E^{(n-1)} + \frac{E_{act}^{(n-1)} - E_{act}^{(n)}}{E_{act}^{(n-1)}}
\]

where \( E_{act}^{(n-1)} \) is the input energy calculated at previous lift-off. Selection of this update scheme is done in the same way as it was done for energy loss parameter \( K_{loss} \). As seen in Fig. 9, the energy offset is completely eliminated. Such learning controllers are possible because of the cyclic nature of the motions that provide the opportunity for the robot to “learn from its past errors.”

### B. Planar Running Control

1) Locomotion Time: The hip motion exhibits a periodic motion that must be in tune with the vertical hopping motion that plays the role of a “pace maker.” The leg angle has to be at its desired value when touchdown happens, no matter what the touchdown time happens to be. Therefore, a locomotion time that takes a fixed values at touchdown or lift-off is a better variable for trajectory expressions. Options for locomotion time were introduced in [1] that resulted in more robust performance. Two conditions need to be satisfied for a valid locomotion time \( \eta \). First, it should be a scalar valued function which maps one flight phase onto the interval \( E = (-1, +1) \) with \( \eta_{td} = -1 \) at lift-off, \( \eta_{td} = +1 \) at touchdown, and \( \eta = 0 \) at apex. Second, \( \eta \) is an affine function of time. For the stance phase, although not as critical as the flight, the requirement stays the same except that \( \eta \) varies from +1 to -1 from touchdown to lift-off. With these two conditions, \( \eta \) becomes a “time-like” parameter suitable for motion planning with a more natural synchronization.

As mentioned before, the lift-off and touchdown heights depend on the leg actuator position at those instances, \( p_{td,lo} \) and \( p_{td,hi} \). The robot motion during flight is almost ballistic, starting with touchdown height \( z_{td} = r_0 + p_{td,lo} \), and ending with \( z_{lo} = r_0 + p_{td,lo} \). The only difficulty in solving this motion is that final actuator position during flight \( p_{td,lo} \) may not be known beforehand. However, during flight the actuator is always retracted to a fixed position \( p_{lo} \). In that case, the flight duration can be evaluated by

\[
T_f = \frac{z_{lo} + \sqrt{z_{lo}^2 - 2g(p_{lo} - p_{td,lo})}}{g}, \quad z_{lo}^2 > 2g(p_{lo} - p_{td,lo})
\]

and the touchdown velocity by

\[
\dot{z}_{td} = -\sqrt{z_{lo}^2 - 2g(p_{lo} - p_{td,lo})},
\]

The velocity profile is linear in time during flight, with known boundaries \( z \in (z_{lo}, z_{td}) \). Assuming time \( t \) is zero at lift-off, \( \dot{z} = \ddot{z}_{lo} - gt \). A simple locomotion time \( \eta_l = (0, T_f) \leftrightarrow (-1, 1) \) can take the form

\[
\eta_l(t) = \frac{2t - (z_{lo} + z_{td})}{z_{td} - z_{lo}} = -\frac{22}{z_{td} - z_{lo}} \cdot t - 1.
\]

It can be verified that \( \eta_l(0) = -1 \) and \( \eta_l(T_f) = +1 \). For the planar case, the locomotion can be adjusted by

\[
\dot{z}_{td} = -\sqrt{z_{lo}^2 - 2g(p_{lo} - p_{td,lo}) \cos(\theta_{td}) - p_{td,lo} \cos(\theta_{td,lo})}.
\]

Equation (22) accounts for height change due to the desired touchdown leg angle \( \theta_{td,lo} \) and the lift-off leg angle \( \theta_{lo} \). Indeed, using (22) made very minor effects. The maximum touchdown angle has been recorded to be \( 8^\circ \) at the top running speed of 1.25 m/s. In that case, \( \cos(\theta_{td}) \approx 0, \theta \), which confirms the small difference between (20) and (22).

During stance, the locomotion time \( \eta_s(t) : (-T_s/2, T_s/2) \leftrightarrow (1, -1) \) is defined by

\[
\eta_s = \frac{2}{T_s} t_s, \quad \eta_s(t_{td}) = -1, \quad \eta_s(t_{lo}) = 1
\]

where \( T_s \) is the stance period measured from the last step. The variations of \( \eta_s \) and \( \eta_t \) shows a good degree of precision and linearity, as depicted in Fig. 10. The truncation of the stance graphs is due to coarse recording and high speeds at touchdown.

2) Forward Speed Control: The robot has to adapt its relative speed to the treadmill to maintain its position at the center. Let the treadmill speed be \( \dot{z}_{tread} \) and \( \dot{x} \) be the absolute velocity of the robot, then the relative speed of the robot with respect to the treadmill will be \( \dot{z}_{rel} = \dot{x} + \dot{z}_{tread} \). The objective is to maintain \( x_{td} = \dot{z}_{td} = 0 \). Raibert’s foot placement algorithm was adopted to control the forward velocity

\[
x_{ft,td} = x^{*}_{ft,td} + K_x(x - x_{td}) + K_{\dot{x}}(\dot{x} - \dot{x}_{td})
\]

\[
= x^{*}_{ft,td} + K_x x + K_{\dot{x}} \dot{x}
\]

where \( x^{*}_{ft,td} \) is the neutral foot position, \( K_x \) and \( K_{\dot{x}} \) are the gains, and \( \dot{z}_{tread} \) is the average relative velocity. The neutral touchdown foot position \( x^{*}_{ft,td} \) can be found via...
The trajectories are used by a controller to follow the desired leg angles. At the higher level, the controller calculates the required control input \( p_{\text{rsd}} \) as determined by (29). Small friction forces from planarizer guides, small change of center of mass due to hip actuator motion, toe being hinged to the ground during stance, small damping forces, and linear hip and leg springs, are among the assumptions made. The stance equation of motion for the body simplifies to

\[
\ddot{\phi} = \frac{Rk_h \left[ p_{\text{rsd}} + R(\theta - \phi) \right] - m_h g z_{\text{opt}} \sin(\phi)}{J_h \cdot \phi_{\text{des}} + K_{\text{hf}} \theta_{\text{des}} + K_{\text{pd}} \dot{\phi}_{\text{des}}},
\]

where

\[
p_{\text{rsd}} = -R(\theta - \phi) - \frac{J_l}{Rk_h} \left[ \dot{\theta}_d + K_\lambda \dot{\theta}_d + K_{\text{ps}} \dot{\theta}_d \right]
\]

\[
\dot{p}_{\text{rsd}} = -R(\ddot{\theta} - \ddot{\phi}) - \frac{J_l}{Rk_h} \left[ \dot{\theta}_d^{(3)} + K_{\text{fr}} \dot{\theta}_d + K_{\text{ps}} \dot{\theta}_d \right]
\]

\[
\ddot{p}_{\text{rsd}} = -R(\dddot{\theta} - \dddot{\phi}) - \frac{J_l}{Rk_h} \left[ \dot{\theta}_d^{(4)} + K_{\text{fr}} \dot{\theta}_d^{(3)} + K_{\text{ps}} \dot{\theta}_d \right]
\]

(27)

The constants \( k_h \) and \( r_h \) are the hip spring stiffness and ball screw lead, the \( K \)'s are the gains, \( J_l \) the leg inertia about the hip, and \( \alpha_h = J_s / r_h^2 \) is a constant in terms of the combined motor and screw inertia \( J_s \).

4) Body Pitch Tracking Control (Stance): The desired body pitch angle for the stance is given by

\[
\phi_{\text{des}}(\eta) = \hat{\phi}_{\text{des}} \sin(\pi \rho \eta),
\]

(28)

In the above, \( \eta \) is the stance locomotion time, and \( \hat{\phi}_{\text{des}} = -J_h / J_{\text{pd}} \) is the desired amplitude for body pitch oscillations. Again, a simplified model is used in the stance controller. Small friction forces from planarizer guides, small change of center of mass due to hip actuator motion, toe being hinged to the ground during stance, small damping forces, and linear hip and leg springs, are among the assumptions made. The stance equation of motion for the body simplifies to

\[
\ddot{\phi} = \left( \frac{Rk_h \left[ p_{\text{rsd}} + R(\theta - \phi) \right] - m_h g z_{\text{opt}} \sin(\phi)}{J_h \cdot \phi_{\text{des}} + K_{\text{hf}} \theta_{\text{des}} + K_{\text{pd}} \dot{\phi}_{\text{des}}} \right) / J_h.
\]

A similar two-level controller is used to track \( \phi_{\text{des}}(\eta) \). The actuator controller is the same as the one built for the flight (26), while the reference position is found via

\[
p_{\text{rsd}} = -R(\theta - \phi) + \frac{m_h g z_{\text{opt}} \sin(\phi)}{Rk_h} \cdot \frac{J_h}{J_{\text{pd}}} \cdot \phi_{\text{des}} + K_{\text{hf}} \theta_{\text{des}} + K_{\text{pd}} \dot{\phi}_{\text{des}}\]

(30)

with \( \epsilon_{\phi} = \phi_{\text{des}} - \phi \) and \( \epsilon_{\theta} = \theta_{\text{des}} - \theta \). The angles \( \phi \) and \( \theta \) for a typical experiment shown in Fig. 12 confirm a steady-state counter oscillation with an amplitude ratio of 4.8, which is relatively close to the designed ratio of \( J_h / J_l = 0.75/0.14 = 5.4 \), despite all the losses and approximations of the real run.

It is important to study the robot’s behavior both in transients and steady state. The transients demonstrate the stability and robustness of the controller under varying input velocities, while the steady state gives a better insight into the energetics. In a typical robot experiment in Fig. 13, the robot starts with vertical hopping (\( J_{\text{tread}} = 0 \)), then ramps up and maintains its maximum speed of 1 m/s, and finally returns to zero speed. Fig. 14 shows that the vertical controller performs very well in controlling the energy (upper plot) and the hopping height (lower plot) as expected. Detailed discussion of the energetics in steady state are left for Section V.
Fig. 11. Schematic of the forward speed controller.

Fig. 12. Leg-body counter oscillations. "o" is for touchdown and "x" for lift-off.

Fig. 13. Experimental data of apex height and forward velocity. Height accuracy of ±5 mm and forward position accuracy of ±30 m were achieved.

Fig. 14. Top: desired (dashed) and actual (solid) added energy. Bottom: the hopping height. "o" is for touchdown and "x" for lift-off.

TABLE III

ENERGETICS OF THE ROBOT AVERAGED OVER SEVERAL STEPS

<table>
<thead>
<tr>
<th>Robot</th>
<th>Motor</th>
<th>Stance Phase</th>
<th>Flight Phase</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monopod I</td>
<td>Leg</td>
<td>Thrust: 13 J</td>
<td>Retraction: 12 J</td>
<td>25 J</td>
</tr>
<tr>
<td></td>
<td>Hip</td>
<td>Pitch: 5 J</td>
<td>Leg Angle: 20 J</td>
<td>25 J</td>
</tr>
<tr>
<td>Monopod II</td>
<td>Leg</td>
<td>Thrust: 5 J</td>
<td>Retraction: 5.5 J</td>
<td>10.5 J</td>
</tr>
<tr>
<td></td>
<td>Hip</td>
<td>Pitch: 3.7 J</td>
<td>Leg Angle: 6.5 J</td>
<td>10.2 J</td>
</tr>
</tbody>
</table>

V. ENERGETICS ANALYSIS

The primary motivation for adding compliance at the hip joint was its potential for energy saving. After having described how the ensuing control challenges were solved, this section is devoted to studying the effects of implementing CPDR on the energetics of the ARL-Monopod II and comparing them with those of ARL-Monopod I.

A conservative measure of the mechanical energy expenditure is the integral of the absolute value of the instantaneous “shaft power”

\[ E = \int_{t_0}^{t_f} |\tau_0| dt \]

where \( \tau \) and \( \omega \) are the torque and the angular velocity of the motor. This measure is conservative, as it does not account for the gained energy from the system (negative work). The mechanical energy supplied by the actuators is predicted to be actually smaller. We found it useful to examine the individual contributions of the hip and leg motors during flight and stance \( E_{h,f}, E_{h,s}, E_{l,f}, \) and \( E_{l,s} \). The total energy expenditure of the robot is the sum of these four components. Any recorded energy values is reset to zero at the beginning of each phase (or step), so that we can only look at the energy per phase. The two limits of the integral can be \( t_{sd} \) and \( t_{fd} \) for the stance, or \( t_{ld} \) and \( t_{fd} \) for the flight. For example, the hip motor energy for the flight is calculated via

\[ E_{h,f} = \int_{t_{ld}}^{t_{fd}} |\tau_0| \omega_0| dt. \]

Note that this measure does not include the motor efficiency of converting input electric power to shaft power, but includes everything from the motor shaft on.

A. Energetics at Constant Speed

The energetics of ARL-Monopod II are summarized in Table III numerically, and in Figs. 15 and 16 graphically. Compared with the ARL-Monopod I [13], where the hip
controller consumed 5 J during stance and 20 J during flight (at 1.2 m/s), the energy savings are significant. The saving amounts to approximately 67% for the flight and 26% for stance. For both flight and stance, the leg actuator savings are just above 50%. The total energy has dropped from 50 J/step to less than 20 J/step, accounting for 58% savings. Considering that the ARL-Monopod II is also heavier (18 kg) than the ARL-Monopod I (15 kg), the improvement in efficiency is even higher.

B. Specific Resistance

A proper measure of efficiency for robots should account for the weight and velocity of the robot, as well as the energetics. A general such measure, named “specific resistance,” was introduced by Gabrielli and von Kármán [34], defined by

\[ \varepsilon = \frac{P}{mg \bar{v}_{\text{max}}} \]

where \( P \) is the output power, \( mg \) the total weight, \( \bar{v}_{\text{max}} \) the maximum speed. This measure with a slight change is used to express \( \varepsilon \) as a function of speeds

\[ \varepsilon(\dot{x}) = \frac{P(\dot{x})}{mg\dot{x}}. \]

(31)

The total output power for ARL-Monopod II, at maximum speed, is \( P = 20.7 \, \text{J/step} \cdot 2.3 \, \text{(steps/s)} \approx 48 \, \text{W} \). Comparing specific resistance for ARL-Monopod II and Monopod I.

\[ \varepsilon_{\text{II}} = \frac{48 \, \text{W}}{18 \, \text{kg} \cdot 9.81 \, \text{m/s}^2 \cdot 1.25 \, \text{m/s}} \approx 0.22 \]

\[ \varepsilon_{\text{I}} = \frac{125 \, \text{W}}{15 \, \text{kg} \cdot 9.81 \, \text{m/s}^2 \cdot 1.2 \, \text{m/s}} \approx 0.7 \]

reveals an overall improvement of 70% in efficiency.

C. Forward Velocity Effects

The required power of the robot has an increasing trend with forward velocity, hence the specific resistance shows an opposite trend. The leg energy [Fig. 17(a)] shows a moderate increase at lower speeds, but a higher rate is observed at higher speeds. The hip energy per step increases almost linearly with velocity, as illustrated in Fig. 17(b). This behavior matches the trend of the simulation results of passive dynamics running, as reported in [1]. Fig. 17(c) shows the total energy expenditure of the system per step. The curve for specific resistance \( \varepsilon \) [Fig. 17(d)] shows a very low efficiency (high \( \varepsilon \)) at low speeds, due to the “hotel load” or the energy spent to maintain the vertical motion at zero speed \( P(0) \). The \( \varepsilon \) curve is almost flat for speeds between 0.9 and 1.25 m/s. This can be attributed to practical limitations such as increased damping forces which are proportional to the square of the velocity.

A practical model for \( \varepsilon \) can be found, assuming that power loss is of the form \( P(\dot{x}) = \alpha \dot{x}^2 + \beta \dot{x} \), where \( \alpha (\text{N}) \) and \( \beta (\text{N/s/m}) \) are the constants. This relationship is consistent with the fact that leg energy fits well by a quadratic curve and hip energy by a line (see Fig. 17). Such a model also matches the formula given for the human mechanical energy expenditure in [35]. Consequently, specific resistance can be expressed by

\[ \varepsilon(\dot{x}) = \frac{P(0) + \alpha \dot{x}^2 + \beta \dot{x}}{mg \dot{x}}, \]

(32)

The minimum value of this curve is at \( \dot{x}_{\text{opt}} = \sqrt{\frac{P(0)}{\alpha}} \), found by setting \( d\varepsilon/d\dot{x} = 0 \). Since \( P(0) > 0 \) and \( \alpha > 0 \), the minimum value always exists and is \( \varepsilon_{\text{opt}} = \varepsilon(\dot{x}_{\text{opt}}) = 2(\sqrt{\alpha P(0)} + \beta)/mg \).

The overhead power for the ARL-Monopod II is \( P(0) = 15.6 \, \text{W} \), but since the experiments did not go beyond 1.25 m/s, we did not observe the minimum \( \varepsilon \). The trend of the curve in Fig. 17 shows that we are operating close to the minimum value estimated to happen at \( \dot{x}_{\text{opt}} = 1.4 \, \text{m/s} \) with \( \varepsilon_{\text{opt}} \approx 0.21 \), from the curve (d) in Fig. 17. Thus, \( \alpha \approx 8.0, \beta \approx 14.8, \) and

\[ \varepsilon(\dot{x}) = \frac{15.6(W) + 8(N \cdot s/m) \cdot \dot{x}^2 + 14.8(N) \cdot \dot{x}}{18(\text{kg}) \cdot 9.81(\text{m/s}^2) \cdot \dot{x}}. \]

(33)
This parametrization suggests some interesting insights. At low speeds, our robot’s $\frac{P}{P_0}$ is dominated by the constant term in the numerator, the total hotel load $P_0$. Indeed, this constant term causes a decreasing trend in $\frac{P}{P_0}$ (increase in efficiency), observed in virtually all reports of robots’ $\frac{P}{P_0}$ as a function of speed in the literature. A notable exception are the (completely unpowered) passive walkers, which show a monotonic increase in $\frac{P}{P_0}$ with speed, thanks to the absence of a hotel load constant term in (33). Fig. 18 compares the specific resistance variations of the ARL-Monopods and other locomotors as a function of forward velocity. ARL-Monopod’s performance at its highest speeds is close to human energetics. Cornell Walker [36] has recently been developed to achieve efficient bipedal walking. With 13 kg weight, its specific mechanical energy expenditure is at 0.055, while its overall specific resistance is 0.2 for a walking gait. Cornell Walker only exhibits a walking gait, while ARL-Monopods exhibit only running gait. McGeer’s “gravity walker” [16] is an efficient and unpowered 3.5 kg mechanism that exhibits a stable walk on an inclined surface driven entirely by gravity.

The upper and lower curves for the energetics of human walking and running reported by Cavagna [35] and Margaria [37] are presented in Fig. 18. Cavagna added the mechanical work required to accelerate and decelerate the limbs, during a constant-speed walk or run, to the variations of kinetic and potential energy of the trunk to obtain the total mechanical energy. The velocity and acceleration data were extracted from high-speed camera images and cinematographical analysis. This measurement uses estimated mass properties of limbs and neglects the effects of elastic energy stored in the muscles and bones resulting in higher energetics than the real expenditure. Margaria used force plate data to directly estimate the horizontal and vertical accelerations to estimate the energetics of running and walking which resulted in lower energetics. Finally, a measurement of the total electrical power consumption (in addition to the mechanical power-based energetics) is not available for ARL-Monopod II experiments. Calculation of efficiency based on total electric power consumption has become the norm for (electrically actuated) robot efficiency reporting since the ARL-Monopod II experiments.

VI. FUTURE RESEARCH

The CPDR strategy could be applied to bipedal running robots. This should be rather straightforward if the leg mor-
phology is pogo-stick-like as in the ARL-Monopods, but more challenging otherwise. Future research would also extend the CPDR framework to quadrupedal running, at challenging otherwise. Future research would also extend the CPDR framework to quadrupedal running, at challenging otherwise. Future research would also extend the CPDR framework to quadrupedal running, at challenging otherwise. Future research would also extend the CPDR framework to quadrupedal running, at challenging otherwise. Future research would also extend the CPDR framework to quadrupedal running, at challenging otherwise. Future research would also extend the CPDR framework to quadrupedal running, at challenging otherwise.

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REFERENCES


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