1. (8 pts) Below is shown a randomly polarized laser beam striking a window of refractive index \( n = 1.6 \) at Brewster’s angle.

![Sketch of Brewster's angle and polarization](image)

a) Label Brewster’s angle on the sketch above and calculate its value below.

\[
\theta_B = \tan^{-1} \frac{\theta_T}{\theta_L} = \tan^{-1} \frac{1.6}{1.0} = 58.0^\circ
\]

b) Show on the sketch above the reflected and transmitted beams, at their correct angles, and indicate their states of polarization.

\[
\theta_T = \sin^{-1} \left( \frac{n_1 \sin \theta_B}{n_2} \right) = \sin^{-1} \left( \frac{1}{1.6} \sin 58^\circ \right) = 32.0^\circ
\]

2. (8 pts) A 1 cm diameter Si PIN photodetector is used to detect a uniform 900 nm wavelength LED beam of intensity 10 \( \mu \text{W/cm}^2 \). Assume the quantum efficiency is 90%, the bias voltage is 12 V, and the load resistor is 10 k\( \Omega \). What is the expected signal \( V_{\text{out}} \) across the load resistor?

\[
P_{\text{opt}} = I \times \text{area} = 10 \frac{\mu \text{W}}{\text{cm}^2} \times (\pi) (0.5 \text{ cm})^2 = 7.854 \mu \text{W}
\]

\[
I_{\text{ph}} = \frac{8 \eta P_{\text{opt}} \lambda_{\text{ph}}}{hc(\text{eV} \cdot \mu \text{m})} = \frac{\eta P_{\text{opt}} \lambda_{\text{ph}}}{1.24 \text{ V}} = \frac{(0.9)(1.900 \mu \text{m})(7.854 \mu \text{W})}{1.24 \text{ V}}
\]

\[
I_{\text{ph}} = 5.13 \mu \text{A}
\]

\[
V_{\text{ph}} = I_{\text{ph}} \times R_L = (5.13 \mu \text{A})(10 \text{ k\( \Omega \)}) = 51.3 \text{ mV}
\]

b) If the load resistor is reduced to 1 k\( \Omega \), how will the signal \( V_{\text{out}} \) change? Explain.

Since \( V_{\text{out}} = I_{\text{ph}} \times R_L \), \( V_{\text{out}} \) will decrease in amplitude by 10x.

\[
R_L = 2.2 \text{ k\( \Omega \)} \quad \text{total will also decrease}
\]

So \( BW = \frac{0.35}{2.2} \) will increase by 10x.
3. (22 pts) An unknown gas laser (assume n = 1) is emitting from a “black box” in a TEM\textsubscript{oo} mode into a scanning Fabry-Perot interferometer as shown below. The scanning FPI output from the detector is also shown below on the oscilloscope trace. The known quantities are labeled on the figure.

![Diagram of Fabry-Perot interferometer](image)

a) What is the nominal wavelength of the laser?

\[ \lambda_0 = \frac{c}{v} = \frac{3 \times 10^{10} \text{ cm/s}}{6.15 \times 10^{14} \text{ Hz}} = \frac{4880 \text{ Å}}{488 \text{ nm}} \]

b) What are \( L_{\text{FP}} \) and \( L_{\text{LASER}} \)?

\[ \text{FSR}_F = \frac{c}{2 n L_{\text{FP}}} = 1500 \text{ MHz} \]
\[ L_{\text{LASER}} = \frac{5 \text{ units}}{1 \text{ unit}} \times L_{\text{FP}} = 50 \text{ cm} \]
\[ L_{\text{FP}} = \frac{3 \times 10^{10} \text{ cm/s}}{2 (1.5 \times 10^{-7} \text{ cm/s})} = 10 \text{ cm} \]

c) How far is the FPI scanning? Explain.

10 units total, \( \lambda / 2 \) repeat distance = 5 units \( \Rightarrow \) Full FPI scan = \( \lambda_0 = 488 \text{ nm} \)

d) What is the finesse of the FPI? Explain

\[ F = \frac{\text{FSR}_F}{\Delta \nu_{\text{FP}}} = \frac{1500 \text{ MHz}}{50 \text{ MHz}} = 30 \]

e) Is the laser homogeneously or inhomogenously broadened? Explain.

Inhomogeneously \( \Rightarrow \) since 2 independent longitudinal modes are lasing.

f) “If” the wavelength of this laser was 1.0 \( \mu \text{m} \) (which it is not), what would be its bandwidth \( \Delta \lambda \) if \( \Delta \nu \) is 1 MHz?

\[ \Delta \lambda = \frac{\lambda^2}{c} \Delta \nu = \frac{(10^{-4} \text{ cm})^2}{3 \times 10^{10} \text{ cm/s}} \times 10^6 \text{ Hz} = \frac{1}{3} \times 10^{-12} \text{ cm} = 0.33 \text{ Å} \]
4. (22 pts) You are required to design a new optical resonator for the new “Beaver” laser with a wavelength of 600 nm (ORANGE color). The cavity length is to be 0.5 m and will use two curved mirrors with equal radii of curvature. The minimum beam radius (spot size) is to be 400 μm.

a) Specify the required radius of curvature of the two laser mirrors.

\[ R_1 = R_2 = R(25 cm) = 25 cm \left( 1 + \left( \frac{25 cm}{25 cm} \right)^2 \right) = 305.7 \text{ cm} \]

\[ Z_R = \frac{\pi (0.4 x 10^{-4} \text{ cm})^2}{\lambda_0} = 93.8 \text{ cm} \]

b) Sketch the cavity and show the location of \( z = 0 \) and the spot size shape of the beam inside and outside the cavity.

c) What is the far-field half divergence angle \( \theta_{1/2} \)?

\[ \Theta_{1/2} = \frac{\lambda_0}{\pi w_0} = \frac{600 \text{ nm}}{\pi (400 \text{ μm})} \]

\[ = 0.477 \text{ mrad} \]

d) Where will the minimum beam phase front radius of curvature occur? Show it on your sketch in (b) above.

\[ R_{min} = 2Z_R \text{ and well equal } R_{min} = R(z_R) = 2Z_R \]

e) The beam is to be a TEM\(_{00}\) beam. Describe the power distribution of this beam and sketch its cross-sectional beam pattern.

\[ I(r) = I_0 e^{-\frac{r^2}{w_0^2}} \]

\[ \text{TEM}_{00} \text{ Gaussian} \]

e) If one of the two mirrors is 100% reflecting, the other mirror is 95% reflecting, there are no losses in the cavity other than the output, and the gain length is 10 cm, what must be the minimum value of gain in cm\(^{-1}\) for the laser to reach threshold?

\[ l = R_1 R_2 e^{2\gamma L} = (1.0)(0.95) e^{2 \gamma_{th} 10 \text{ cm}} \]

\[ \frac{1}{20 \text{ cm}^2} \frac{1}{0.95} = \gamma_{th} = 0.00256 / \text{cm} \]