Problem 2.32

From

\[ \delta[n - k] * h[n] = h[n - k] \]
\[ (ax_1[n] + bx_2[n]) * h[n] = ax_1[n] * h[n] + bx_2[n] * h[n] \]

given that \( h[n] = \delta[n + 1] + 3\delta[n] + 2\delta[n - 1] - \delta[n - 2] + \delta[n - 3] \)

(a) \( x[n] = 3\delta[n] - 2\delta[n - 1] \)

\[ y[n] = x[n] * h[n] \]
\[ = \{ 3\delta[n] - 2\delta[n - 1] \} * h[n] \]
\[ = 3h[n] - 2h[n - 1] \]
\[ = 3\delta[n + 1] + 9\delta[n] + 6\delta[n - 1] - 3\delta[n - 2] + 3\delta[n - 3] - 2\delta[n - 4] - 4\delta[n - 2] + 2\delta[n - 3] - 2\delta[n - 4] \]
\[ = 3\delta[n + 1] + 7\delta[n] - 7\delta[n - 2] + 5\delta[n - 3] - 2\delta[n - 4] \]

(b) \( x[n] = u[n + 1] - u[n - 3] = \delta[n + 1] + \delta[n] + \delta[n - 1] + \delta[n - 2] \)

\[ y[n] = x[n] * h[n] \]
\[ = \{ \delta[n + 1] + \delta[n] + \delta[n - 1] + \delta[n - 2] \} * h[n] \]
\[ = h[n + 1] + h[n] + h[n - 1] + h[n - 2] \]
\[ = \delta[n + 2] + 4\delta[n + 1] + 6\delta[n] + 5\delta[n - 1] + 5\delta[n - 2] + 2\delta[n - 3] + \delta[n - 5] \]

(c) \( x[n] \) as given in Fig. P2.32 (b)

\[ x[n] = 2\delta[n - 3] + 2\delta[n] - \delta[n + 2] \]
\[ y[n] = 2h[n - 3] + 2h[n] - h[n + 2] \]
\[ = -\delta[n + 3] - 3\delta[n + 2] + 7\delta[n] + 3\delta[n - 1] + 8\delta[n - 3] + 4\delta[n - 4] - 2\delta[n - 5] + 2\delta[n - 6] \]

2.38. An LTI system has impulse response \( h(t) \) depicted in Fig. P2.38. Use linearity and time invariance to determine the system output \( y(t) \) if the input \( x(t) \) is

(a) \( x(t) = 2\delta(t + 2) + \delta(t - 2) \)

\[ y(t) = 2h(t + 2) + h(t - 2) \]

(b) \( x(t) = \delta(t - 1) + \delta(t - 2) + \delta(t - 3) \)

\[ y(t) = h(t - 1) + h(t - 2) + h(t - 3) \]

(c) \( x(t) = \sum_{p=0}^{\infty} (-1)^p \delta(t - 2p) \)

\[ y(t) = \sum_{p=0}^{\infty} (-1)^p h(t - 2p) \]
The plot of the $y(t)$ for part (a),

![Plot for part (a)](image1)

The plot of the $y(t)$ for part (b),

![Plot for part (b)](image2)

The plot of the $y(t)$ for part (c),

![Plot for part (c)](image3)

Problem 2.40

(a) $m(t) = x(t) * y(t) = y(t) * x(t) = \int_{-\infty}^{\infty} y(\tau) x(t - \tau) d\tau$
Now we consider the value of \( m(t) \) over different range of time \( t \) by moving \( x(t-\tau) \).

**Case 1:** When \( t + 1 < 0 \), this implies that 
\( t < -1 \)
Therefore, \( m(t) = 0 \)

**Case 2:** When \( 0 < t + 1 < 2 \), this implies that 
\( -1 \leq t < 1 \)
Therefore, 
\[
m(t) = \int_{0}^{t+1} d\tau = t + 1
\]

**Case 3:** When \( 2 < t + 1 < 4 \), this implies that 
\( 1 \leq t < 3 \)
Therefore, 
\[
m(t) = \int_{t-1}^{2} d\tau + \int_{2}^{t+1} 2d\tau = t + 1
\]

**Case 4:** When \( t - 1 < 4, t + 1 > 4 \), this implies that 
\( 3 \leq t < 5 \)
Therefore, 
\[
m(t) = \int_{t-1}^{4} 2d\tau = 10 - 2t
\]
Case 5: When $t-1 > 4$, this implies that $t \geq 5$

Therefore, $m(t) = 0$

Finally, we can say that

$$m(t) = \begin{cases} 
0 & t < -1 \\
(t + 1) & -1 \leq t < 1 \\
(t + 1) & 1 \leq t < 3 \\
10 - 2t & 3 \leq t < 5 \\
0 & t \geq 5 
\end{cases}$$

$$(g) \quad m(t) = y(t) * g(t) = g(t) * y(t) = \int_{-\infty}^{\infty} g(\tau) y(t-\tau) d\tau$$

Now we consider the value of $m(t)$ over different range of time $t$ by moving $y(t-\tau)$.

Case 1: When $t < -1$, $m(t) = 0$
Case 2: When $-1 < t < 1$, this implies that

$$-1 \leq t < 1$$

Therefore,

$$m(t) = \int_{-1}^{t} \tau d\tau = 0.5[t^2 - 1]$$

Case 3: When $t-2 < 1$ and $1 \leq t$, this implies that

$$1 \leq t < 3$$

Therefore,

$$m(t) = \int_{-1}^{t-2} 2\tau d\tau + \int_{t-2}^{1} \tau d\tau = 0.5(t-2)^2 - 0.5$$

Case 4: When $t-4 < 1$ and $1 \leq t-2$, this implies that

$$3 \leq t < 5$$

Therefore,

$$m(t) = \int_{t-4}^{1} 2\tau d\tau = 1 - (t-4)^2$$

Case 5: When $t-4 \geq 1$, this implies that

$$t \geq 5$$

Therefore, $m(t) = 0$

Finally, we can say that

$$m(t) = \begin{cases} 
0 & t < -1 \\
0.5[t^2 - 1] & -1 \leq t < 1 \\
0.5(t-2)^2 - 0.5 & 1 \leq t < 3 \\
1 - (t-4)^2 & 3 \leq t < 5 \\
0 & t \geq 5 
\end{cases}$$
(h) \[ m(t) = y(t) \ast c(t) = \int_{-\infty}^{\infty} y(\tau) c(t-\tau) d\tau \]

Now we consider the value of \( m(t) \) over different range of time \( t \) by moving \( c(t-\tau) \).

**Case 1:** When \( t+2 < 0 \), this implies that

\[ t < -2 \]

Therefore, \( m(t) = 0 \)

**Case 2:** When \( t+2 < 2 \), \( t+2 \geq 0 \) this implies that

\[ -2 \leq t < 0 \]

Therefore, \( m(t) = 1 \) ( from \( \delta(\cdot) \) at \( \tau = t+2 \) )

**Case 3:** When \( t < 1 \), \( t+2 \geq 2 \) this implies that

\[ 0 \leq t < 1 \]

Therefore,

\[ m(t) = \int_{0}^{t} d\tau + 2 = t + 2 \]

**Case 4:** When \( t < 2 \), \( t \geq 1 \) this implies that

\[ 1 \leq t < 2 \]

Therefore,

\[ m(t) = \int_{t-1}^{t} d\tau + 2 = 3 \]
Case 5: When $t-1 < 2$, $t \geq 2$, this implies that
\[ 2 \leq t < 3 \]
Therefore,
\[ m(t) = -1 + \int_{t-1}^{2} d\tau + 2 \int_{2}^{t} d\tau = t - 2 \]

Case 6: When $t < 4$, $t - 1 \geq 2$, this implies that
\[ 3 \leq t < 4 \]
Therefore,
\[ m(t) = -1 + 2 \int_{t-1}^{t} d\tau = 1 \]

Case 7: When $t - 1 < 4$, $t \geq 4$, this implies that
\[ 4 \leq t < 5 \]
Therefore,
\[ m(t) = -2 + 2 \int_{t-1}^{4} d\tau = 8 - 2t \]

Case 8: When $t - 2 < 4$, $t - 1 \geq 4$ this implies that
\[ 5 \leq t < 6 \]
Therefore,
\[ m(t) = -2 \]

Case 9: When $t - 2 \geq 4$, this implies that
\[ t \geq 6 \]
Therefore, $m(t) = 0$
Finally, we can say that

\[
m(t) = \begin{cases} 
0 & t < -2 \\
1 & -2 \leq t < 0 \\
t + 2 & 0 \leq t < 1 \\
3 & 1 \leq t < 2 \\
t - 2 & 2 \leq t < 3 \\
1 & 3 \leq t < 4 \\
8 - 2t & 4 \leq t < 5 \\
-2 & 5 \leq t < 6 \\
0 & t \geq 6 
\end{cases}
\]

**Problem 2.41**

[Refer to textbook, page 67, for the impulse response \( h(t) \) of an RC circuit.]

(a) Use convolution to calculate the received signal due to transmission of a single “1” at time \( t = 0 \). Note that the received waveform extends beyond time \( T \) and into the interval allocated for the next bit, \( T < t < 2T \). This contamination is called intersymbol interference (ISI), since the received waveform at any time is interfered with by previous symbols.

The impulse response of an RC circuit is \( h(t) = \frac{1}{RC} e^{-\frac{t}{RC}} u(t) \). The output of the system is the convolution of the input, \( x(t) \) with the impulse response, \( h(t) \).

Thus,

\[
y_p(t) = h(t) \ast p(t) = p(t) \ast h(t) = \int_{-\infty}^{\infty} p(\tau) h(t - \tau) d\tau = \int_{-\infty}^{\infty} p(\tau) \frac{1}{RC} e^{-\frac{(t-\tau)}{RC}} d\tau
\]

Now we consider the value of \( y_p(t) \) over different range of time \( t \).

**Case 1:** When \( t < 0 \), \( y_p(t) = 0 \)
Case 2: When $t < T$, this implies that

$$0 \leq t < T$$

Therefore,

$$y_p(t) = \int_0^t \frac{1}{RC} e^{-\frac{(t-\tau)}{RC}} d\tau$$

$$y_p(t) = 1 - e^{-\frac{t}{RC}}$$

Case 3: When $t \geq T$, we have

$$y_p(t) = \int_0^T \frac{1}{RC} e^{-\frac{(t-\tau)}{RC}} d\tau$$

$$y_p(t) = e^{-\frac{(t-T)}{RC}} - e^{-\frac{t}{RC}}$$

Finally, we can say that

$$y_p(t) = \begin{cases} 
0 & t < 0 \\
1 - e^{-\frac{t}{RC}} & 0 \leq t < T \\
e^{-\frac{(t-T)}{RC}} - e^{-\frac{t}{RC}} & t \geq T
\end{cases}$$

(b) Use convolution to calculate the received signal due to transmission of the sequences “1110” and “1000”. Compare the received waveforms to the output of an ideal channel ($h(t) = \delta(t)$) to evaluate the effect of ISI for the following choices of $RC$:

(i) $RC = 1/T$

(ii) $RC = 5/T$

(iii) $RC = 1/(5T)$

Assuming $T = 1$

sequences “1110”

$$x(t) = p(t) + p(t-1) + p(t-2) - p(t-3)$$

$$y(t) = y_p(t) + y_p(t-1) + y_p(t-2) - y_p(t-3)$$
sequences “1000”

\[
\begin{align*}
(2) \quad x(t) &= p(t) - p(t - 1) - p(t - 2) - p(t - 3) \\
y(t) &= y_p(t) - y_p(t - 1) - y_p(t - 2) - y_p(t - 3)
\end{align*}
\]
2.42. Use the definition of the convolution sum to prove the following properties

(a) Distributive: \( x[n] * (h[n] + g[n]) = x[n] * h[n] + x[n] * g[n] \)

\[
\text{LHS} = x[n] * (h[n] + g[n]) \\
= \sum_{k=-\infty}^{\infty} x[k] \cdot (h[n-k] + g[n-k]) : \text{The definition of convolution.} \\
= \sum_{k=-\infty}^{\infty} (x[k]h[n-k] + x[k]g[n-k]) : \text{the dist. property of mult.} \\
= \sum_{k=-\infty}^{\infty} x[k]h[n-k] + \sum_{k=-\infty}^{\infty} x[k]g[n-k] \\
= x[n] * h[n] + x[n] * g[n] \\
= \text{RHS}
\]

(b) Associative: \( x[n] * (h[n] * g[n]) = (x[n] * h[n]) * g[n] \)

\[
\text{LHS} = x[n] * (h[n] * g[n]) \\
= x[n] * \left( \sum_{k=-\infty}^{\infty} h[k]g[n-k] \right) \\
= \sum_{l=-\infty}^{\infty} x[l] \cdot \left( \sum_{k=-\infty}^{\infty} h[k]g[n-k-l] \right) \\
= \sum_{l=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} (x[l]h[k]g[n-k-l]) \\
\text{Use } v = k + l \text{ and exchange the order of summation} \\
= \sum_{v=-\infty}^{\infty} \left( \sum_{l=-\infty}^{\infty} x[l]h[v-l] \right) g[n-v] \\
= \sum_{v=-\infty}^{\infty} (x[v] * h[v]) g[n-v] \\
= (x[n] * h[n]) * g[n] \\
= \text{RHS}
\]

(c) Commutative: \( x[n] * h[n] = h[n] * x[n] \)
\[ \text{LHS} = x[n] * h[n] \]
\[ = \sum_{k=-\infty}^{\infty} x[k]h[n - k] \]

Use \( k = n - l \)
\[ = \sum_{l=-\infty}^{\infty} x[n - l]h[l] \]
\[ = \sum_{l=-\infty}^{\infty} h[l]x[n - l] \]
\[ = \text{RHS} \]

2.46. Find the expression for the impulse response relating the input \( x[n] \) or \( x(t) \) to the output \( y[n] \) or \( y(t) \) in terms of the impulse response of each subsystem for the LTI systems depicted in

(a) Fig. P2.46 (a)
\[ y(t) = x(t) * \{ h_1(t) - h_4(t) \ast [h_2(t) + h_3(t)] \} \ast h_5(t) \]

(b) Fig. P2.46 (b)
\[ y[n] = x[n] \ast \{-h_1[n] \ast h_2[n] \ast h_4[n] + h_1[n] \ast h_3[n] \ast h_5[n]\} \ast h_6[n] \]

(c) Fig. P2.46 (c)
\[ y(t) = x(t) \ast \{[-h_1(t) + h_2(t)] \ast h_3(t) \ast h_4(t) + h_2(t)\} \]