1. Let
\[ x(t) = \sum_{k=-\infty}^{\infty} [3\delta(t-k) - \delta(\frac{t}{3}-k)] \]  

(a) Carefully plot \( x(t) \) with proper labels. (5 pts)
(b) Is \( x(t) \) periodic? If so, find its fundamental period. If not, explain why? (10 pts)
(b) From a, we could see that $x(t)$ is periodic with fundamental period 3.

2. A time-discrete system $H$ is described by the following input-output relationship:

$$y[n] = \sum_{i=0}^{n+2} \frac{1}{2} x[i]$$

(a) Is $H$ time-invariant? (10 pts)

(b) Is $H$ BIBO stable? (10 pts)

(a)

Let $x_i[n] = x[n - N]$

$$y_i[n] = \sum_{i=0}^{n+2} \frac{1}{2} x_i[i]$$

$$= \frac{1}{2} \sum_{i=0}^{n+2} x[i - N]$$

$$= \frac{1}{2} \sum_{i=-N}^{n+2} x[i]$$

$$y[n - N] = \sum_{i=0}^{n-N+2} \frac{1}{2} x[i]$$

$\neq y_i[n]$

$\Rightarrow$ The system is time variant.

(b)

Suppose the bounded input: $x[n] = 1$ for all $n$

$$|y[n]| = \sum_{i=0}^{n+2} \frac{1}{2} |x[i]| = \frac{1}{2} (n + 3)$$

As $|y[n]|$ is unbounded $\Rightarrow$ the system is not BIBO
3. Given an input \( x(t) \) and a time-continuous LTI system with the impulse response \( h(t) \) shown in Fig. 1, find and carefully sketch the output \( y(t) = x(t) * h(t) \). Make sure you label all the points carefully (40 pts).

![Figure 1: Problem 3.](image)

Method 1:

+) \( t < -1 \)

\[ y(t) = 0 \]

+) \( -1 \leq t < 0 \)

\[ y(t) = \int_{-1}^{\tau} (\tau + 1) d\tau = \left( \frac{\tau^2}{2} + \tau \right) \bigg|_{-1}^{t} = \frac{t^2}{2} + t + \frac{1}{2} \]

+) \( 0 \leq t < 1 \)

\[ y(t) = \int_{0}^{t} (\tau + 1) d\tau + \int_{0}^{\frac{1}{2}} \tau d\tau = \frac{t^2}{2} + \frac{1}{2} \]

+) \( 1 \leq t < 2 \)

\[ Y(t) = \int_{1}^{\frac{1}{2}} \tau d\tau = -\frac{t^2}{4} + \frac{t}{2} \]

+) \( 2 \leq t \)

\[ y(t) = 0 \]
Method 2:

$h_0(t)$ is a single triangle function

$y_0(t) = x(t) * h_0(t)$

+ $t < 0 \Rightarrow y_0(t) = 0$
+ $0 \leq t < 1$

$y_0(t) = \int_0^t \tau d\tau = \left[ \frac{\tau^2}{2} \right]_0^t = \frac{t^2}{2}$

+ $1 \leq t < 2$

$y_0(t) = \int_{t-1}^1 \tau d\tau = \left[ \frac{\tau^2}{2} \right]_{t-1}^1 = \frac{1 - (t - 1)^2}{2}$

+ $2 \leq t \Rightarrow y_0(t) = 0$

$h(t) = h_0(t+1) + \frac{1}{2} h_0(t)$

$y(t) = y_0(t+1) + \frac{1}{2} y_0(t)$

+ $t < -1 \Rightarrow y(t) = 0$
+ $-1 \leq t < 0 \Rightarrow y(t) = \frac{(t+1)^2}{2} = \frac{t^2}{2} + t + \frac{1}{2}$
+ $0 \leq t < 1 \Rightarrow y(t) = \frac{1-(t+1-1)^2}{2} + \frac{1}{2} t^2 = \frac{t^2}{4} + \frac{1}{2}$
+ $1 \leq t < 2 \Rightarrow y(t) = \frac{1}{2} \frac{1-(t-1)^2}{2} = -\frac{t^2}{4} + \frac{1}{2}$
+ $2 \leq t \Rightarrow y(t) = 0$
4. A time-discrete LTI system $H$ with the input $x[n]$ produces the output $y[n]$ as shown in Fig. 2(a). Plot the output $w[n]$ of $H$ corresponding to the input $z[n]$ as shown in Fig. 2(b). (25 pts)

![Figure 2: Problem 4.](image)

$x[n] = 2\delta[n] + \delta[n-1]$

$z[n] = 2\delta[n] - \delta[n-1] - \delta[n-2]$

$= 2\delta[n] + \delta[n-1] - 2\delta[n-1] - \delta[n-2]$

$= x[n] - x[n-1]$

As it is a LTI system, thus

$w[n] = y[n] - y[n-1]$

$= \delta[n-1] + \delta[n-2] - (\delta[n-2] + \delta[n-3])$

$= \delta[n-1] - \delta[n-3]$
5. **(Bonus question (10pts))** Find a particular solution \( y_p[n] \) to the following difference equation:

\[
2y[n] - 3y[n - 1] = n;
\]

From the Table 2.3 (textbook), we know,

For input \( x[n] = n \), the particular solution should be in the form \( y_p[n] = c_1 n + c_2 \).

Put it back to the original difference equation,

\[
2y_p[n] - 3y_p[n-1] = n
\]

\[
2(c_1 n + c_2) - 3(c_1(n-1) + c_2) = n
\]

\[
2c_1 n + 2c_2 - 3c_1 n + 3c_1 - 3c_2 = n
\]

\[
-c_1 n + 3c_1 - c_2 = n
\]

\[
\begin{align*}
-c_1 &= 1 \\
3c_1 - c_2 &= 0
\end{align*}
\]

\[
\begin{align*}
&\Rightarrow \begin{cases}
  c_1 = -1 \\
  c_2 = -3
\end{cases} \\
&\Rightarrow y_p[n] = -n - 3
\end{align*}
\]