1. (35 pts) A certain reverberation effect can be modeled using the following LTI system with the following frequency response:

\[ H(j\omega) = \frac{1 - \omega}{1 + \omega^2} \]  

(a) Let \( x(t) = 2 + \sin^2\pi t \), write \( x(t) \) as a linear combination of complex exponentials, i.e., determine \( c_i \) and \( w_i \) so that \( x(t) = \sum_c c_i e^{jw_i t} \).

(b) Using the result in (a) and the property of an LTI system with an input as a linear combination of complex exponentials, determine the output \( y(t) \).

\[ x(t) = 2 + \sin^2\pi t = 2e^{j0t} + \left( \frac{e^{j\pi t} - e^{-j\pi t}}{2j} \right)^2 = 2e^{j0t} - \frac{e^{j2\pi t} - 2 + e^{-j2\pi t}}{4} = \left(2 + \frac{1}{2}\right)e^{j0t} - \frac{1}{4}e^{j2\pi t} - \frac{1}{4}e^{-j2\pi t} \]

\[ y(t) = \sum_{i=0}^{2} c_i e^{j\omega_i t} \]

\[ c_0 = \frac{5}{2}; \ w_0 = 0; \ c_1 = -\frac{1}{4}; \ w_1 = 2\pi; \ c_2 = -\frac{1}{4}; \ w_2 = -2\pi; \]

\[ y(t) = \sum_{i=1}^{2} H(j\omega_i) c_i e^{j\omega_i t} = \frac{5}{2} \cdot \frac{1}{4} \cdot \frac{1}{1 + 4\pi^2} e^{j2\pi t} - \frac{1}{4} \cdot \frac{1 + 2\pi}{4} e^{-j2\pi t} \]

\[ y(t) = \sum_{i=1}^{2} H(j\omega_i) c_i e^{j\omega_i t} = \frac{5}{2} \cdot \frac{1}{4} \cdot \frac{1 - 2\pi}{1 + 4\pi^2} e^{j2\pi t} - \frac{1}{4} \cdot \frac{1 + 2\pi}{4} e^{-j2\pi t} \]
2. (35 pts) Let a system with the following impulse response

\[ h(t) = e^{-|t|} \]  

(a) Using the definition of Fourier transform, show that \( H(j\omega) = \frac{2}{1 + j\omega^2} \)

(b) Let \( x_1(t) = 1 \) and \( x_2(t) = \sin 100\pi t \). Let \( z(t) = h(t)(x_1(t) + x_2(t)) \). Without performing any convolution or Fourier transformation, can you tell whether \( z(t) \) contain more energy from \( x_1(t) \) or \( x_2(t) \)? In other words, which of the signals \( x_1(t) \) or \( x_2(t) \) has been suppressed in the output \( z(t) \)? Justify your answer.

\[ a) \quad H(jw) = \int_{-\infty}^{\infty} e^{-|t|} e^{-j\omega t} dt = \int_{-\infty}^{0} e^t e^{-j\omega t} dt + \int_{0}^{\infty} e^{-t} e^{-j\omega t} dt = \frac{e^{(1-j\omega)t}}{(1-j\omega)} \bigg|_{-\infty}^{0} + \frac{e^{-(1-j\omega)t}}{-(1-j\omega)} \bigg|_{0}^{\infty} = \]

\[ \frac{1}{1 - j\omega} + \frac{1}{1 + j\omega} = \frac{2}{1 + \omega^2} \]

\[ b) \]

\( H(j\omega) \) is a low pass filter so \( x_1 \) has the most effect on the answer this filter passes the low frequency signal.

3. (30 pts) Let

\[ x[n] = \sum_{m=0}^{\infty} \left(-\frac{1}{2}\right)^m \delta[n-2m] \]  

(a) Sketch \( x[n] \) for \( n = -1, 0, 1, 2, 3, 4 \)

(b) Determine the appropriate Fourier representation of \( x[n] \).

(a) 

(b) 

\( x[n] \) is discrete and nonperiodic signal, so use DTFT
\[ X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n} \]
\[ = \sum_{n=-\infty}^{\infty} \left( \frac{-1}{2} \right)^m \sum_{m=0}^{\infty} \delta[n - 2m] e^{-j\Omega n} \]
\[ = \sum_{m=0}^{\infty} \left( \frac{-1}{2} \right)^m \sum_{m=0}^{\infty} \delta[n - 2m] e^{-j\Omega n} \]
\[ = \sum_{m=0}^{\infty} \left( \frac{-1}{2} \right)^m e^{-2j\Omega m} \]
\[ = \frac{1}{1 + \frac{1}{2} e^{-2j\Omega}} \]

4. (10pts) Bonus question. Let \( x(t) = 2 + \cos(2\pi t) \)

Determine the appropriate Fourier representation of \( x(t) \).

\( x(t) \) is continuous and periodic function

\[ 2\pi T = 2k\pi \text{ k is integer number.} \]

\[ T = k \]

So the fundamental period is 1;

\[ \omega_0 = \frac{2\pi}{T} \]
\[ = 2\pi \]

\[ x(t) = 2 + \cos(2\pi t) \]
\[ = 2 + \frac{1}{2} (e^{j2\pi t} + e^{-j2\pi t}) \]
\[ = 2e^{0\times j2\pi t} + \frac{1}{2} e^{1\times j2\pi t} + \frac{1}{2} e^{(-1)\times j2\pi t} \]
\[ = \sum_{k=-\infty}^{\infty} X[k] e^{jk2\pi t} \]

So
\[ X[k] = \begin{cases} 
2 & , k = 0 \\
1 & , k = \pm 1 \\
2 & , o.w. \\
0 & 
\end{cases} \]