1. a.

\[ V_1 = z_{11} I_1 + z_{12} I_2 \]

\[ V_2 = -R I_2 = z_{21} I_1 + z_{22} I_2 \rightarrow I_2 = \frac{-z_{21} I_1}{z_{22} + R} \]

\[ V_1 = I_1 \left[ z_{11} - \frac{z_{12} z_{21}}{z_{22} + R} \right] = z_{11} I_1 \]

\[ z_{11} = \frac{z_{11}^2 - z_{12}^2 + R z_{11}}{z_{11} + R} = R \]

\[ z_{12}^2 - z_{12}^2 = R^2 = (z_{11} + z_{12})(z_{11} - z_{12}) \]

\[ z_{11} = \frac{z_A + z_B}{2} \quad z_{21} = z_{11} \left( \frac{z_B - z_A}{z_B + z_A} \right) = \frac{z_B - z_A}{2} \]

\[ R^2 = z_A z_B = \frac{S L}{\mu C} = \frac{1}{C} \rightarrow R = \sqrt{\frac{1}{C}} = 500 \Omega \]

\[ z_{11} = \frac{s^2 LC + 1}{2\pi C} = z_{22} \]

\[ z_{12} = \frac{1 - s^2 LC}{2\pi C} = z_{21} \]

\[ z_{11}, z_{12} = \frac{1 \pm 4 \times 10^{-18} s^2}{8 \times 10^{-12} \Omega} \]

b. By symmetry, \( S_{22} = 0 \).

Since both ports are matched, and twoport is lossless, \( |S_{12}| = |S_{21}| \equiv 1 \).
\[ Z_{11} = Z_{22} = \frac{Z_A + Z_B}{2} \]
\[ Z_{12} = Z_{21} = \frac{Z_B - Z_A}{Z_B + Z_A} \]
\[ V_1 = \frac{E_1}{2} = \frac{E_1}{2R} \left( Z_{11} - Z_{12} \right) \frac{V_2}{R} \]
\[ V_2 = \frac{Z_{12}}{R} = \frac{E_1}{2R} \left( Z_{11} - R \right) \]
\[ S_{21} = \frac{V_2}{E_{1/2}} = \frac{Z_{11} - R}{Z_{12}} = \frac{Z_A + Z_B - 2R}{Z_B - Z_A} = S_{12} \]
\[ = \frac{sL + 1/sC - \sqrt{1/C}}{1/sC - sL} = \frac{s^2LC - 2s\sqrt{LC} + 1}{1 - s^2LC} \]
\[ S_{12} = \frac{(s\sqrt{LC} - 1)^2}{(1 + s\sqrt{C})(1 - s\sqrt{C})} = \frac{1 - sT}{1 + sT} \quad T = 2ns \]
\[ |S_{21}(j\omega)| = \left| \frac{1 - j\omega T}{1 + j\omega T} \right| = 1 \quad \forall \omega \]
SOLUTION To find $h_{11}$, we leave port 2 short-circuited and evaluate the input impedance at port 1.

$$h_{11} = \left. \frac{E_1}{I_1} \right|_{E_2=0} = 100 + \frac{1}{10^{-3} + j2\pi \times 10^6 \times 10^{-10}}$$

$$= 100 + \frac{10^3}{1 + j0.6283} = 817.0 - j450.5 \Omega$$

To find $h_{12}$, we leave port 1 open and find the voltage ratio between port 1 and port 2. We readily see that

$$h_{12} = \left. \frac{E_1}{E_2} \right|_{I_1=0} = 0$$

To find $h_{21}$, we short-circuit port 2 and find the current ratio between port 2 and port 1, or

$$h_{21} = \left. \frac{I_2}{I_1} \right|_{E_2=0} = \frac{0.1}{10^{-3} + j2\pi \times 10^6 \times 10^{-10}}$$

$$= \frac{100}{1 + j0.6283} = 71.7 - j45.05$$

To find $h_{22}$, we short-circuit port 1 and evaluate the admittance seen at port 2. Since $E_1 = 0$, $I_1 = 0$, and $E = 0$, we have

$$h_{22} = \left. \frac{I_2}{E_2} \right|_{I_1=0} = \frac{1}{50 \times 10^3} = 2 \times 10^{-5} \text{ U}$$