1. A Butterworth filter has been designed for a passband ripple of 0.15 dB and a minimum stopband attenuation of 45 dB. Now it needs to be redesigned for a new stopband ripple of 0.10 dB, with the same order and selectivity. What will be the new value of the minimum stopband loss?

2. a. Find the adjoint network of a gyrator with resistance $R$. Express the sensitivity to a change in $R$ in terms of the gyrator currents in the physical and adjoint networks.

        b. Find the sensitivities of the output current $j_{out}$ to variations of the terminations and the gyration resistance $R$ in the circuit shown.

![Circuit Diagram](image)

3. The two-ports shown below are equivalent. Find

        a. the impedances $Z_a$, $Z_b$, and $Z_c$;

        b. the scattering matrix at $\omega_t = 500$ Mra/s, when the circuit is terminated by two 50 ohm resistors.

![Circuit Diagram](image)
1. A Butterworth filter has been designed for a passband ripple of 0.15 dB and a minimum stopband attenuation of 45 dB. Now it needs to be redesigned for a new stopband ripple of 0.10 dB, with the same order and selectivity. What will be the new value of the minimum stopband loss?

\[
A_{s2}^2 = 45 - 1.77 = 43.23 \text{ dB}
\]
2. a. Find the adjoint network of a gyrator with resistance $R$. Express the sensitivity to a change in $R$ in terms of the gyrator currents in the physical and adjoint networks.

b. Find the sensitivities of the output current $j_{out}$ to variations of the terminations and the gyration resistance $R$ in the circuit shown.

And the corresponding adjoint network

From the circuit,

\[ j_1 = j_2 = \frac{e - V_2}{R_1} = \frac{0 - 25}{100} = -\frac{1}{4} \]  

\[ j_3 = j_4 = \frac{-V_3}{R_4} = \frac{-25 \cdot \frac{1}{4}}{50} = -\frac{5}{32} \]
\[ J_3 = \frac{S_2}{2} = \frac{1}{2} \left( \frac{1}{100} - \frac{1}{4} \right) \]

\[ 2J_3 = \frac{1}{100} - \frac{S_2}{4} \]

\[ S_3 = \frac{1}{225} A \]

\[ \Rightarrow J_4 = -J_3 = -\frac{1}{225} A \]

\[ J_2 = 2J_3 = \frac{2}{225} A = \hat{J}_2 \]

From the adjoint network,

\[ \hat{J}_1 = \hat{J}_2 = \frac{\hat{U}_2}{R_1} = \alpha \hat{S}_3 = \frac{25 \hat{S}_3}{100} = \frac{1}{4} \hat{J}_3 \]

\[ \Rightarrow \hat{J}_3 = \hat{J}_4 = \frac{\hat{U}_2 - \hat{U}_3}{R_4} = \frac{\hat{E}_2 - \alpha \hat{S}_2}{R_4} = -\frac{1}{50} \hat{S}_2 = -\frac{1}{50} - \frac{1}{2} \hat{S}_2 \]

\[ \Rightarrow \hat{J}_2 = \frac{1}{4} \left( -\frac{1}{50} - \frac{1}{2} \hat{S}_2 \right) \]

\[ 4.5 \hat{J}_2 = -\frac{1}{50} \]

\[ \hat{S}_2 = -\frac{1}{225} A = \hat{J}_1 \]

\[ \hat{J}_3 = 4 \hat{J}_2 = -\frac{4}{225} A = \hat{J}_4 \]

To find the sensitivities,

\[ \frac{\delta S_0}{\delta R_1} = -\hat{J}_1 \hat{J}_1 = -\left( \frac{1}{225} \right) \left( \frac{2}{225} \right) = \frac{2}{50625} \text{ A/\Omega} \]

\[ \frac{\delta S_0}{\delta R_4} = -\hat{J}_4 \hat{J}_4 = -\left( \frac{4}{225} \right) \left( \frac{1}{225} \right) = \frac{4}{50625} \text{ A/\Omega} \]
\[
\frac{\delta i_o}{\delta \alpha} = + \vec{J}_3 \cdot \vec{j}_2 = \vec{j}_2 \cdot \vec{j}_3 = + \left( -\frac{4}{225} \right) \left( -\frac{225}{225} \right) - \left( -\frac{1}{225} \right) \left( \frac{1}{225} \right)
\]

\[
\text{change in direction of } \alpha = \frac{7}{50625} \text{ A/}^\circ
\]

Also,

\[
\begin{align*}
\Delta R_{l_{\text{max}}} &= 100 \times 0.1 = 10 \Omega \\
\Delta R_{q_{\text{max}}} &= 50 \times 0.1 = 5 \Omega \\
\Delta \chi_{\text{max}} &= 25 \times 0.1 = 2.5 \Omega
\end{align*}
\]

\[
\Rightarrow \left| \Delta j_0 \right|_{\text{max}} = \left| \frac{\delta j_o}{\delta R_l} \Delta R_{l_{\text{max}}} \right| + \left| \frac{\delta j_o}{\delta R_q} \Delta R_{q_{\text{max}}} \right| + \left| \frac{\delta j_o}{\delta \chi} \Delta \chi_{\text{max}} \right|
\]

\[
= \left( \frac{2}{50625} \times 10 \right) + \left( \frac{4}{50625} \times 5 \right) + \left( \frac{7}{50625} \times 2.5 \right)
\]

\[
= 1.1358 \times 10^{-3} \text{ A}
\]

\[
= 1.136 \text{ mA}
\]

\[
\therefore \text{ the range of } j_0 \text{ is } \pm 1.1358 \text{ mA}
\]

\[
\begin{align*}
\bar{j}_o &= -\frac{1}{225} \text{ A} = -4.44 \text{ mA} \\
\hat{j}_o &= -4.44 \pm 1.136 \text{ mA}
\end{align*}
\]

\[
\Rightarrow \text{ The range of } j_0 \text{ is}
\]

\[
\boxed{5.58 \text{ mA} \leq j_0 \leq 3.31 \text{ mA}}
\]
3. The two-ports shown below are equivalent. Find

a. the impedances $Z_a$, $Z_b$, and $Z_c$;

b. the scattering matrix at $\omega_1 = 500 \text{ Mra/s}$, when the circuit is terminated by two 50 ohm resistors.
\[ z_{12} = \frac{1}{2} \left( \frac{1}{sL} + \frac{1}{sC} \right) = \frac{1}{sLC} = Z_C \]
\[ z_{11} = \frac{1}{2} \left( \frac{s}{2L} + \frac{1}{sC} \right) = \frac{sL + 1}{sLC} = Z_L + Z_C \]
\[ z_a = \frac{sL}{sC} = Z_L \]

At \( 500 \text{ Ma/s} = \omega_1 \),
\[ \frac{1}{\omega_1 C} = \frac{1}{(5 \times 10^8 \times 4 \times 10^{-11})} = 125 \text{ ohm} \]

So the lattice becomes

\[ S_{22} = \frac{\frac{1}{50} - \frac{50 (2+j)}{50 + 100 (2+j)}}{100 + 100 j} = \frac{-1}{1+j} = \frac{-1+j}{2} \]
\[ S_{21} = S_{12} = \frac{-2Z_L}{50 + Z_L} = \frac{-2100}{50 (2+j) + j50} = \frac{-1+j}{2} \]
\[ V_n = V_{22} = \frac{1}{1 + \frac{d}{1 + \frac{d}{2}}} = \frac{1 + \frac{d}{2}}{1 + d} \]

\[ V_{12} = V_{21} = V_n = \frac{\frac{d}{2}}{1 + d} = \frac{\frac{d}{2}}{1 + d} \]

\[ S_{11} = S_{22} = 1 - 2V_{11} = 1 - \frac{2 + d}{1 + d} = -1 + \frac{d}{1 + d} = \frac{1 + d}{2} \]

\[ S_{12} = S_{21} = -2V_{12} = -\frac{d}{1 + d} = \frac{1 + \frac{d}{2}}{2} \]

*Note*: Lattice becomes

\[ L = 0.1 \mu F \]

Causing \( S_{12} \) and \( S_{21} \) to change sign.