Transducer Function

\[ H(s) = \frac{1}{S_{21}} = \frac{1}{2} \sqrt{\frac{R_1}{R_2}} \frac{E_{11}(s)}{V_2(s)} \]

\[ |H(j\omega)|^2 = \left( \frac{P_{\text{max},\text{free}}}{P_2} \right)^2 = \frac{1}{|S_{21}(j\omega)|^2} \]

\[ \rho_1(s) = \frac{Z_1 - R_1}{Z_1 + R_1} = S_{11}(s) \]

\[ |\rho_1(j\omega)|^2 = \frac{P_{\text{refl}}}{P_{\text{max,gen}}} \quad \text{with} \quad P_{\text{refl}} = P_{\text{max,gen}} - P_1 \]

\[ H(s), K(s), \rho_1(s) \] are all rational functions.

\[ K(s) = \frac{S_{11}(s)}{S_{21}(s)} = H(s)\rho_1(s) \]

\[ |K(j\omega)|^2 = \frac{P_{\text{refl}}}{P_2} \quad \text{zeros: unity gain frequencies} \]

\[ \text{poles: transmission zeros} \]

If the two-port is lossless \( P_1 = P_{\text{max,gen}} - P_{\text{refl}} = P_2 \)

And \( |H(j\omega)|^2 - |K(j\omega)|^2 = 1 \)

As shown later, this leads to

\[ H(s)H(-s) = K(s)K(-s) + 1 \quad \text{(\(*) \text{ Feldtkeller equation)} \]

From which \( H(s) \) can be obtained. For active two-ports, it may still be convenient to use \( (\*) \), although the power relations are no longer valid.
$C_n$ dimension $(\text{ra}/s)^{-h}$, so $C_n \rightarrow \omega_o^{-h}$

$K(s) = C_n s^n \Rightarrow (s/\omega_o)^n \leq S^h$

$S$: normalized complex freq. variable

$\alpha = 10 \log_{10} \left[ \frac{|s/\omega_o|^2 + 1}{2} \right] \rightarrow 10 \log_{10} 2 \approx 3.96\, \text{dB}$

for $s = j\omega_o$, $\leq 0 \omega_o \equiv 200\, \text{dB}$

In terms of $S$,

$H(s) H(-s) = (-1)^n S^n + 1$.

$H(s)$ must have its zeros in the closed LHP: strictly Hurwitz polynomial.

All coefficients must be $> 0$.

For $n = 2$, $H(s) = a_2 s^2 + a_1 s + a_0$

$(a_2 s^2 + a_1 s + a_0)(a_2 s^2 - a_1 s + a_0) = S^4 + 1$

$a_2 = 1$, $a_0 = 1$

$2a_0 a_2 s^2 - a_1^2 s^2 = 0 \rightarrow a_1 = \sqrt{2}$

For $n = 3$, $a_3 = a_0 = 1$, $a_1 = a_2 = 2$ (show!)

Becomes messy for high $n$. 