Definitions

- **Network** – Any structure containing interconnected elements.
- **Circuit** – Usually physical structure constructed from electrical components.

(A) **Linear Network**: response proportional to excitation. Superposition applies:

\[
\text{If } e_1(t) \rightarrow w_1(t) \text{ and } e_2(t) \rightarrow w_2(t)
\]

Then
\[
k_1 \cdot e_1(t) + k_2 \cdot e_2(t) \rightarrow k_1 \cdot w_1(t) + k_2 \cdot w_2(t)
\]

(B) **Time-Invariant Network**: \( e(t) \rightarrow w(t) \) relation the same if \( t \rightarrow t + t_1 \). Time varying otherwise.

(C) **Passive Network**: EM energy delivered always non-negative. Specifically:

\[
E(t) = \int v(x) i(x) dx \geq 0
\]

or

\[
E(t) = \int_{t_0}^t v(x) i(x) dx + E(t_0) \geq 0
\]

This must be true for any voltage and its resulting current for all \( t \).

Otherwise, active.

(D) **Lossless Circuit**: input energy is always equal to the energy stored in the network. Otherwise, lossy.

(E) **Distributed Network**: must use Maxwell's equation to analyze. Examples: transmission lines, high speed VLSI circuits, \( \mathcal{L} \mathcal{C} \mathcal{E} \).

(F) **Memoryless or Resistivity Circuit**: no energy storing elements. Response depends only on instantaneous excitation. Otherwise, dynamic or memoried circuit.
(G) **Reciprocity:** response remains the same if excitation and response locations are interchanged. Specifically:

- Case (a):
  
  \[ R_{12} \rightarrow R_{21} \]

- Case (b):
  
  \[ P_{12} = 0 \]

- Case (c):
  
  \[ Q_{12} = \infty \]

Otherwise, non-reciprocal:

\[ h_{12} = -h_{21} \]

\[ l_{12} = -l_{21} \]
(H) **Lumped Network**: physical dimensions can be considered zero. In reality, much smaller than the wavelength of the signal.

![Lumped Network Diagram]

\[ k_1 = k_2 \]

\[ \text{KCL} \]

\[ \text{KVL} \]

\[ \text{Boundary conditions} \]

(I) **Continuous-Time Circuit**: the signals can take on any value at any time.

![Continuous-Time Circuit Diagram]

(J) **Sampled-Data Circuit**: the signals have a known value only at some discrete time instances. Digital, analog circuits.

An ideal RLC circuit is linear, time-invariant, passive, lossy, reciprocal, lumped, dynamic continuous-time network.
(A) Ideal R, L, C:  \[ \Rightarrow 0 \]

<table>
<thead>
<tr>
<th>Element</th>
<th>Parameter</th>
<th>Direct</th>
<th>Inverse</th>
<th>Symbol</th>
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</thead>
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<tr>
<td>Resistor</td>
<td>Resistance R</td>
<td>[ v = Ri ]</td>
<td>[ i = \frac{1}{R}v ] = [ Gv ]</td>
<td>[ \uparrow R \downarrow ]</td>
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<tr>
<td></td>
<td>Conductance G</td>
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<tr>
<td>Inductor</td>
<td>Inductance L</td>
<td>[ v = L \frac{di}{dt} ]</td>
<td>[ i(t) - \frac{1}{L} \int_0^t v(x) dx + i(0) ]</td>
<td>[ \uparrow L \downarrow ]</td>
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<td>Inverse Inductance T</td>
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<tr>
<td>Capacitor</td>
<td>Capacitance C</td>
<td>[ i = C \frac{dv}{dt} ]</td>
<td>[ v(t) = \frac{1}{C} \int_0^t i(x) dx + v(0) ]</td>
<td>[ \uparrow C \downarrow ]</td>
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<td>Elastance D</td>
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Table 1

Each passive;

Assuming standard references, the energy delivered to each of the elements starting at a time when the current and voltage were zero will be:

\[ E_R(t) = \int R i^2(x) dx \geq 0 \quad (67) \]

\[ E_L(t) = \int L \frac{di(x)}{dx} i(x) dx = \int_0^i L \frac{di}{dt} \frac{di}{dt} = \frac{1}{2} Li^2(t) \geq 0 \quad (68) \]

\[ E_C(t) = \int C \frac{dv(x)}{dx} v(x) dx = \int v(0) C \frac{dv}{dx} \frac{dv}{dx} = \frac{1}{2} Cv^2(t) \geq 0 \quad (69) \]

\[ \text{Homework} \]

\[ E_C = ? \quad \text{Stored energy} \]

\[ E_V = ? \quad \text{Delivered energy} \]
Ideal Transformer:

\[
\text{(a)} \quad \begin{array}{c}
\text{Ideal} \\
V_1 \\
- \\
\end{array} \quad \begin{array}{c}
i_1 \\
\rightarrow \\
n:1 \\
i_2 \leftarrow \\
\end{array} \quad \begin{array}{c}
\text{Ideal} \\
V_2 \\
- \\
\end{array}
\]

\[
\text{(b)} \quad \begin{array}{c}
\text{Ideal} \\
V_1 \\
- \\
\end{array} \quad \begin{array}{c}
i_1 \\
\rightarrow \\
n:1 \\
\end{array} \quad \begin{array}{c}
\text{Ideal} \\
V_2 \\
- \\
\end{array} \quad \begin{array}{c}
\rightarrow \\
R \\
\leftarrow \\
\end{array}
\]

\[v_2 = -n^2 R \frac{i_2}{i_1}\]

Fig. 6 An ideal transformer

Defined in terms of the following v-i relationships:

\[
v_1 = n v_2 \quad \text{(70a)}
\]

\[
i_2 = -n i_1 \quad \text{(70b)}
\]

or

\[
\begin{bmatrix}
v_1 \\
i_2
\end{bmatrix} = \begin{bmatrix}
0 & n \\
-n & 0
\end{bmatrix} \begin{bmatrix}
i_1 \\
v_2
\end{bmatrix} \quad \text{(70c)}
\]

\[
v_1 = n v_2 = -n R i_2 = (n^2 R) i_1 \quad \text{(71)}
\]

At the input terminals, then, the equivalent resistance is \(n^2 R\). Observe that the total energy delivered to the ideal transformer from connections made at its terminals will be

\[
E(t) = \int (v_1(x)i_1(x) + v_2(x)i_2(x))dx = 0
\]

Lossless, memoryless!

The right-hand side results when the v-i relations of the ideal transformer are inserted in the middle. Thus, the device is passive; it transmits, but neither stores nor dissipates energy.

Memoryless!
(C) **Physical Transformer:**

L₁: primary self-inductance
M: mutual inductance

Fig. 7 A transformer

The diagram is almost the same except that the diagram of the ideal transformer shows the turns ratio directly on it. The transformer is characterized by the following v-i relationships for the reference shown in Fig. 7:

\[ v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \]  \hspace{1cm} (73a)

And

\[ v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt} \]  \hspace{1cm} (73b)

Thus it is characterized by three parameters: the two self-inductances L₁ and L₂, and the mutual inductance M. The total energy delivered to the transformer from external sources is

\[ E(t) = \int [v_1(x)i_1(x) + v_2(x)i_2(x)]dx \]

\[ = \int i_1^2dx_1 + \int i_2^2dx_2 + \int Mdi_1di_2 \]

\[ = \frac{1}{2}(L_1i_1^2 + 2Mi_1i_2 + L_2i_2^2) \geq 0 \hspace{1cm} \text{due to physical considerations!} \]

It is easy to show that the last line will be non-negative if

\[ \frac{M^2}{L_1L_2} = k^2 \leq 1 \]

**Homework 2**

Since physical considerations require the transformer to be passive, this condition must apply. The quantity k is called the *coefficient of coupling*. Its maximum value is unity for a closely-coupled transformer.
A transformer for which the coupling coefficient takes on its maximum value \( k = 1 \) is called a perfect, or perfectly coupled, transformer. A perfect transformer is not the same thing as an ideal transformer. To find the difference, turn to the transformer equations (73) and insert the perfect-transformer condition \( M = \sqrt{L_1 L_2} \); then take the ratio \( v_1/v_2 \). The result will be

\[
\frac{v_1}{v_2} = \frac{\frac{L_1}{dt} \frac{di_1}{dt} + \sqrt{L_1 L_2} \frac{di_2}{dt}}{\sqrt{L_1 L_2} \frac{di_1}{dt} + L_2 \frac{di_2}{dt}} = \sqrt{L_1/L_2}.
\]  

(76)

This expression is identical with \( v_1 = nv_3 \) for the ideal transformer† if

\[
n = \sqrt{L_1/L_2}.
\]  

(77)

Next let us consider the current ratio. Since (73) involve the derivatives of the currents, it will be necessary to integrate. The result of inserting the perfect-transformer condition \( M = \sqrt{L_1 L_2} \) and the value \( n = \sqrt{L_1/L_2} \), and integrating (73a) from 0 to \( t \) will yield, after rearranging,

\[
i_1(t) = -\frac{1}{n} i_2(t) + \left\{ \frac{1}{L_1} \int_0^t v_1(x) \, dx + \left[ i_1(0) + \frac{1}{n} i_2(0) \right] \right\}.
\]  

(78)

This is to be compared with \( i_1 = -i_2/n \) for the ideal transformer. The form of the expression in brackets suggests the \( v-i \) equation for an inductor. The diagram shown in Fig. 8 satisfies both (78) and (76). It shows how a perfect transformer is related to an ideal transformer. If, in a perfect transformer, \( L_1 \) and \( L_2 \) are permitted to approach infinity, but in such a way that their ratio remains constant, the result will be an ideal transformer.

![Diagram](image)

**Fig. 8.** Relationship between a perfect and an ideal transformer.

*Lossless, memory element.*
(D) **The Gyraotor:**

**Definitions:**
- **Port:** Two terminals, both input leads always carrying the same current.
- **Gyraotor:** A two port network requiring active components for realization.

![Fig. 9 A gyraotor](image)

**Time domain analysis**

\[ V_i(t) = \frac{V_1 - V_2}{2} + j \frac{V_1 + V_2}{2} \]

For Fig. 9(a)

\[ V_1 = -r_2 i_1 \quad \text{or} \quad \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 & -r \\ r & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \]  (79a)

For Fig. 9(b)

\[ V_1 = r_1 i_2 \quad \text{or} \quad \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 & r \\ -r & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \]  (79b)

\[ E(t) = \int_{-\infty}^{\infty} (v_1 i_1 + v_2 i_2) dx = \int_{-\infty}^{\infty} [(-ri_2)i_1 + (ri_1)i_2] dx = 0 \]  (80)

**Loadless Memristors Non-reciprocal**

\[ i_2 = -C \frac{dv_2}{dt} \]

Therefore, upon inserting the v-i relations associated with the gyraotor, we observe that

\[ v_1 = -r_2 i_2 = -r \left(-C \frac{dv_2}{dt}\right) = rC \frac{d(r_i)}{dt} = r^2 C \frac{di}{dt} = L \frac{di_i}{dt} \]  (82)

\[ V_1 = 1 \mathrm{mV} \rightarrow V_2 = 1 \mathrm{V} \]

\[ i_1 = 1 \mathrm{mA} \rightarrow V_2 = 1 \mathrm{V} \]

\[ i_2 = 1 \mathrm{mA} \rightarrow V_1 = -1 \mathrm{V} \]

Non-reciprocal
(The first one is more practical, using transconductors)

\( r = R \)

\[ Q \begin{bmatrix} C \\ L \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \]

\[ \frac{3}{2} L = r^2 C \]

**Figure 7-18 Ideal gyrator circuit**

**Figure 7-24 Floating-inductor simulation using gyrator**
The Riordan circuit using two op-amps:

\[ V(s) = \frac{Z}{s} \frac{V(s)}{I(s)} \]

\[ s = j \omega \rightarrow \text{phasor} \]

\[ I_s = \frac{V}{Z_s} \]

G1C

Opamps must be stable, ideal CMRR!

Floating opamps
\[ V_{CM} = V \]
Linearly!

What is \( Z(\omega) \)?

"Homework 4"

Figure 7-19 The Riordan circuit: (a) basic circuit;
(b) used as an inductor; (c) used as a gyrator
A circuit which uses two grounded-output op-amps and is useful for the realization of either GICs or GIIs is shown in Fig. 7-19a. 

The input impedance $Z$ can easily be found, as follows. When we recall that the input voltage of an op-amp is very nearly zero,

$$V \approx V_2 \approx V_4$$  \hspace{1cm} (7-62)

is obtained. Also, if we denote the current through $Z_1$ by $I_1$ (with the reference direction pointing left to right), the current through $Z_2$ by $I_2$, etc., clearly

$$I_1 \approx I \quad V - V_1 = I_1 Z_1 \approx V_2 - V_1 = -I_2 Z_2$$

$$I_3 \approx I_2 \quad V_2 - V_3 = I_3 Z_3 \approx V_4 - V_3 = -I_4 Z_4$$  \hspace{1cm} (7-63)

$$I_5 \approx I_4 \quad V \approx V_5 = I_5 Z_5$$

Here we assumed, as usual, that the current in the input leads of the op-amps is zero.

Working backward in (7-63) leads to

$$V \approx I_5 Z_5 \approx I_4 Z_5 \approx -I_3 Z_4 \approx I_2 Z_3 \approx I_1 Z_2 \approx I Z_1 Z_3 Z_4 Z_5 \approx \frac{Z_1 Z_3 Z_4 Z_5}{Z_5 Z_4}$$  \hspace{1cm} (7-64)

Hence

$$Z = \frac{V}{I} \approx \frac{Z_1 Z_3 Z_4 Z_5}{Z_5 Z_4}$$  \hspace{1cm} (7-65)

If $Z_5$ is regarded as a load impedance, the circuit behaves like a GIC; (7-66) takes the form

$$Z(s) = f(s) Z_5(s) \quad f(s) = \frac{Z_1(s) Z_3(s)}{Z_2(s) Z_4(s)}$$  \hspace{1cm} (7-66)

If, for example, $Z_1 = R_1$, $Z_2 = 1/s C_2$, $Z_3 = R_3$, $Z_4 = R_4$, and $Z_5 = R_5$ (Fig. 7-19b), then $f(s) = R_1 R_3/((1/s C_2) R_4)$ and

$$Z = \frac{R_1 R_3}{(1/s C_2) R_4} = \frac{s}{2} \frac{R_1 C_2 R_3 R_5}{R_4} \approx L$$  \hspace{1cm} (7-67)

Hence, the input impedance is that of an inductor, with an equivalent inductance value $L_{eq} = R_1 C_2 R_3 R_5/R_4$.

As (7-67) suggests, and as can be directly verified from (7-65), the two-port formed by regarding the terminals of $Z_2$ as an output port is a gyrator if all other impedances are purely resistive (Fig. 7-19c). More generally, if the terminals of $Z_5$ (or $Z_1$ or $Z_2$) constitute the output port, the circuit of Fig. 7-19a is a GIC; if the terminals of $Z_2$ (or $Z_4$) form the output port, the resulting two-port is a GII.

Assume now that we choose $Z_2$ and $Z_4$ as capacitive and $Z_1$, $Z_3$, and $Z_5$ as resistive impedances. Then (7-65) gives, for $s = jo \omega$,

$$Z(j\omega) = \frac{R_1 R_3 R_5}{(1/jo C_2)(1/jo C_4)} = -\omega^2 R_1 C_2 R_3 C_4 R_5$$  \hspace{1cm} (7-68)

We note that $Z(j\omega)$ is pure real, negative, and a function of $\omega$. Such an impedance is called a frequency-dependent negative resistance (FDNR). A slightly different form of FDNR can be obtained, e.g., by choosing $Z_1$ and $Z_3$ as capacitors and $Z_2$, $Z_4$, and $Z_5$ as resistors. Then

$$Z(j\omega) = -\frac{R_5}{C_1 R_2 C_4 \omega^2}$$  \hspace{1cm} (7-69)

As we shall see later, FDNRs are very useful for the design of active filters.