ECE 580 Project 1

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Figure 1 - Rauch Filter Circuit

Specifications:

- Pass-band 0 – 20 kHz;
- DC gain = 1;
- 3-dB bandwidth 100 kHz;
- Equal-valued capacitors;
- Maximum thermal pass-band noise 1 µV
Requirements
1. Find $R2/R1$ so that $Q$ is maximized. What is $Q_{max}$?
2. Find all resistances for minimum power dissipation.
3. Find the capacitances.

Abstract steps taken in chronological format
1. Find parameters required to achieve $Q_{max}$ and DC Gain of 1
2. Find resistances based off of noise specification of $1uV^2$
3. Find capacitances based off of $3dB$ frequency

Step 1: Extracting Parameters using $Q_{max}$
It is given that the transfer function for a second order low-pass filter is as follows.

$$H(s) = \frac{A_v\omega_0^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

The transfer function of the Rauch filter is given below.

$$H(s) = \frac{G_3 G_1 G_2}{s^2 + \frac{G_1 + G_2 + G_3}{C_1} s + \frac{G_1 G_2}{C_1 C_2}}$$

By substituting expressions we can determine that,

$$A_v = \frac{G_3}{G_1}$$

$$\omega_0^2 = \frac{G_1 G_2}{C_1 C_2}$$

$$\frac{\omega_0}{Q} = \frac{G_1 + G_2 + G_3}{C_1}$$

Also, the filter will have two real poles or a pair of complex conjugate poles depending on the $Q$. Either way $\omega_0$ will be the geometric mean of the poles.

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

To achieve a DC gain of 1 we set $G_1 = G_3 = G$ and since both capacitors have been specified to be equal we set $C_1 = C_2 = C$. Also, from the hand out $Q = \frac{R_2}{R_1} = \frac{G_1}{G_2} = \alpha > 0$. Plugging these identities into the Rauch filter transfer function we obtain the following,
\[ H(s) = \frac{1 \, G^2}{\alpha \, C^2} \frac{1}{s^2 + \frac{G(2 + \frac{1}{\alpha})}{C} s + \frac{1}{\alpha \, C^2}} \]

From the simplified transfer function we can derive that,

\[ Q(\alpha) = \frac{\sqrt{\alpha}}{2\alpha + 1} \]

and that \( Q_{\text{max}} \) occurs when \( \alpha = \frac{1}{2} \) where \( Q \left( \frac{1}{2} \right) = \frac{1}{2\sqrt{2}} \).

Since \( Q_{\text{max}} < \frac{1}{2} \) the poles are real and will be,

\[ \omega_1 = \frac{G}{C} = \frac{1}{R_1 C_1} \]
\[ \omega_2 = \frac{1 \, G}{\alpha \, C} = \frac{1}{R_2 C_2} \]

Finally, to satisfy the 3dB corner frequency specification we will set \( f_{3dB} = 100kHz \).

**Step 2: Extracting R-Value using Noise Requirement**

Because the 3dB frequency is much larger than the pass-band limit of 20kHz the thermal noise due to the resistors can be calculated by the three following equations:

\[ V_1^2 = 4KTR \times 1^2 \times \Delta f \]
\[ V_2^2 = 4KTR \times \left( \frac{R_1}{R_3} \right)^2 \times \Delta f \]
\[ V_3^2 = 4KTR \times \left( \frac{R_1 + R_3}{R_3} \right)^2 \times \Delta f \]

Where \( V_1, V_2, V_3 \) is the noise due to \( R_1, R_2, R_3 \) respectively and \( \Delta f = 20kHz \).
Total noise is then:

\[ V_1^2 + V_2^2 + V_3^2 = V_{total}^2 = 1 \mu V^2 \]

Solving for R, where R=R1 we find R is 773\( \Omega \) and from the previous sections, we know that R1=R3, R2=R1/2.

\[
\begin{align*}
R1 &= 773\Omega \\
R2 &= 386\Omega \\
R3 &= 773\Omega 
\end{align*}
\]

**Step 3: Extracting Capacitor Value using 3dB frequency**

\[
|H(j\omega_{3dB})| = \frac{1}{\sqrt{2}}
\]

Solving for C when \( R = 773\Omega \) and \( f_{3dB} = 100kHz \) yields \( C = C_1 = C_2 = 1.21nF \).
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>$Q_{max}$</td>
<td>$\frac{1}{2\sqrt{2}}$</td>
</tr>
<tr>
<td>$R_2/R_1$</td>
<td>$\frac{1}{2}$</td>
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<tr>
<td>$R_1$</td>
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