Exam review

CS 261 Lab #10
The exam will be **comprehensive**
(so don’t forget to review the midterm slides!)

Similar style as midterm
(multiple choice, matching, true/false, code)

This review will focus on **trees, heaps, hash tables, and graphs**
Draw a complete binary search tree that contains the following letters:
A, B, C, D, E, F, G, H, I, J, K
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A, B, C, D, E, F, G, H, I, J, K
Draw a **complete binary search tree** that contains the following letters:

A, B, C, D, E, F, G, H, I, J, K
Tree traversals
Tree traversals

Pre-order
10, 5, 1, 8, 7, 6, 34, 56, 40, 60
Tree traversals

Pre-order
10, 5, 1, 8, 7, 6, 34, 56, 40, 60

In-order
Tree traversals

**Pre-order**
10, 5, 1, 8, 7, 6, 34, 56, 40, 60

**In-order**
1, 5, 6, 7, 8, 10, 34, 40, 56, 60
Tree traversals

Pre-order
10, 5, 1, 8, 7, 6, 34, 56, 40, 60

In-order
1, 5, 6, 7, 8, 10, 34, 40, 56, 60

Post-order

Tree traversals

Pre-order
10, 5, 1, 8, 7, 6, 34, 56, 40, 60

In-order
1, 5, 6, 7, 8, 10, 34, 40, 56, 60

Post-order
1, 6, 7, 8, 5, 40, 60, 56, 34, 10
How would we add 13?
How would we add 13?

Larger values go to the right, smaller values go to the left.
How would we add 13?

Larger values go to the right, smaller values go to the left.
How would we remove 10?
How would we remove 10?

Replace with the left-most item of the right-subtree!
How would we remove 10?

Replace with the left-most item of the right-subtree!
How would we remove 10?
Replace with the left-most item of the right-subtree!
Write a `treeSort` function that takes an array of elements and returns those elements in sorted order. Assume you do not have an iterator (your approach must be recursive). You will need a helper function.
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```c
struct AVLTree* newAVLTree();
void addAVLTree(struct AVLTree *tree, TYPE val);

void treeSort (TYPE data[], int n) {
    // WRITE ME
}

void _treeSortHelper(AVLNode *cur, TYPE *data, int *count) {
    // WRITE ME
}
```
```c
void treeSort(TYPE data[], int size){
    int i;
    int sortIdx = 0;
}
```
void treeSort(TYPE data[], int size){
    int i;
    int sortIdx = 0;

    /* declare an AVL tree */
    struct AVLTree *tree = newAVLTree();
    assert(data != NULL && size > 0);
}

void treeSort(TYPE data[], int size){
    int i;
    int sortIdx = 0;

    /* declare an AVL tree */
    struct AVLTree *tree = newAVLtree();
    assert(data != NULL && size > 0);

    /* add elements to the tree */
    for (i = 0; i < size; i++)
        addAVLTree(tree, data[i]);
}

void treeSort(TYPE data[], int size){
    int i;
    int sortIdx = 0;

    /* declare an AVL tree */
    struct AVLTree *tree = newAVLTree();
    assert(data != NULL && size > 0);

    /* add elements to the tree */
    for (i = 0; i < size; i++)
        addAVLTree(tree, data[i]);

    /* call the helper function on the root */
    _treeSortHelper(tree->root, data, &sortIdx);
}

void _treeSortHelper(AVLNode *cur, TYPE *data, int *index) {

}"
/* *index goes from 0 to size-1 */

void _treeSortHelper(AVLNode *cur, TYPE *data, int *index){
    /* In-order traversal: get the left subtree, then this node, then the right subtree */
    if (cur != NULL) {
        _treeSortHelper(cur->left, data, index);
        data[*index] = cur->val;
        (*index)++;
        _treeSortHelper(cur->right, data, index);
    }
}
Is the height of any binary search tree with \( n \) nodes always \( O(\log n) \)?
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No, unbalanced trees may have height \( n \).
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No, unbalanced trees may have height \( n \)

Does inserting into an AVL tree with \( n \) nodes require looking at \( O(\log n) \) nodes?
Is the height of any binary search tree with \( n \) nodes always \( O(\log n) \)?

No, unbalanced trees may have height \( n \)

Does inserting into an AVL tree with \( n \) nodes require looking at \( O(\log n) \) nodes?

Yes, because AVL trees are balanced
Is the height of any binary search tree with $n$ nodes always $O(\log n)$?

No, unbalanced trees may have height $n$

Does inserting into an AVL tree with $n$ nodes require looking at $O(\log n)$ nodes?

Yes, because AVL trees are balanced

Does inserting into an AVL tree with $n$ nodes require $O(\log n)$ rotations?
Is the height of any binary search tree with $n$ nodes always $O(\log n)$?

No, unbalanced trees may have height $n$

Does inserting into an AVL tree with $n$ nodes require looking at $O(\log n)$ nodes?

Yes, because AVL trees are balanced

Does inserting into an AVL tree with $n$ nodes require $O(\log n)$ rotations?

No, we need at most 2 rotations
Add 12 to this AVL tree
Add 12 to this AVL tree
Add 12 to this AVL tree

Now 10 is unbalanced on right, need to rotate left
Add 12 to this AVL tree

Now 10 is unbalanced on right, need to rotate left
Remove 3
Remove 3

Still balanced, no rotations needed
Remove 3

Still balanced, no rotations needed
Remove 5
Remove 5
Remove 5
Remove 5

Now 7 is heavier on the right (14), which is heavier on the left. Need a **double rotation**...
So we rotate 14 right...
Remove 5 (continued)

So we rotate 14 right...
So we rotate 14 right...
Remove 5 (continued)

... and then rotate 7 left!
Remove 5 (continued)

... and then rotate 7 left!
Remove 5 (continued)

... and then rotate 7 left!
When do we need to do a double rotation?
When do we need to do a double rotation?

1) The node is unbalanced and
When do we need to do a **double rotation**?

1) The node is unbalanced **and**

2) The node’s balance factor is positive, but its right subtree’s balance factor is negative, **or**

The node’s balance factor is negative, but its left subtree’s balance factor is positive
When do we need to do a **double rotation**?

1) The node is unbalanced **and**

2) The node’s balance factor is positive, but its right subtree’s balance factor is negative, **or**

The node’s balance factor is negative, but its left subtree’s balance factor is positive

Balance factor = height(right subtree) - height(left subtree)
How do we represent a binary heap?
How do we represent a binary heap?

An array
How do we represent a binary heap?

An array

What are the indices for the children of node $i$?
How do we represent a binary heap?

An array

What are the indices for the children of node $i$?

$2 \times i + 1$ and $2 \times i + 2$
How do we represent a binary heap?

An array

What are the indices for the children of node $i$?

$2 \times i + 1$ and $2 \times i + 2$

What is the index of the parent of node $i$?
How do we represent a binary heap? 

An array

What are the indices for the children of node $i$?

$2 \times i + 1$ and $2 \times i + 2$

What is the index of the parent of node $i$?

$(i - 1) / 2$
How do we represent a binary heap?

**An array**

What are the indices for the children of node $i$?

$2 \times i + 1$ and $2 \times i + 2$

What is the index of the parent of node $i$?

$(i - 1) / 2$

How do we add a node to a heap?
How do we represent a binary heap?

An array

What are the indices for the children of node $i$?

$2 \times i + 1$ and $2 \times i + 2$

What is the index of the parent of node $i$?

$(i - 1) / 2$

How do we add a node to a heap?

Insert it after the last item, then percolate it up
Simulate **heap sort** on this heap
Simulate **heap sort** on this heap

We’ll work this out on the chalk board
If we have a hash table with 11 buckets and the silly hash function $\text{hash}(x) = x$, what will the hash table look like after inserting 3, 43, 8, 11, 14, 25, 23, 44 using open addressing?
If we have a hash table with 11 buckets and the silly hash function $hash(x) = x$, what will the hash table look like after inserting 3, 43, 8, 11, 14, 25, 23, 44 using open addressing?
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```
bucket = hash(x) % 11
```

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If we have a hash table with 11 buckets and the silly hash function \( \text{hash}(x) = x \), what will the hash table look like after inserting 3, 43, 8, 11, 14, 25, 23, 44 using open addressing?

\[
\begin{array}{c|c}
\text{bucket} & \text{value} \\
\hline
0 & \text{empty} \\
1 & \text{empty} \\
2 & \text{empty} \\
3 & 3 \\
4 & \text{empty} \\
5 & \text{empty} \\
6 & \text{empty} \\
7 & \text{empty} \\
8 & \text{empty} \\
9 & \text{empty} \\
10 & 43 \\
\end{array}
\]
If we have a hash table with 11 buckets and the silly hash function $hash(x) = x$, what will the hash table look like after inserting 3, 43, 8, 11, 14, 25, 23, 44 using open addressing?

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\[
\text{bucket} = \text{hash}(x) \mod 11
\]

If 'bucket' is in use, try the next one.

- **Bucket 0**: 11
- **Bucket 2**: 3
- **Bucket 8**: 43
If we have a hash table with 11 buckets and the silly hash function \( \text{hash}(x) = x \), what will the hash table look like after inserting 3, 43, 8, 11, 14, 25, 23, 44 using **open addressing**?

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\text{bucket} = \text{hash}(x) \mod 11
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$\text{bucket} = \text{hash}(x) \mod 11$

If ‘bucket’ is in use, try the next one.
If we have a hash table with 11 buckets and the silly hash function $hash(x) = x$, what will the hash table look like after inserting 3, 43, 8, 11, 14, 25, 23, 44 using **open addressing**?

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</tbody>
</table>
```

`bucket = hash(x) % 11`

If ‘bucket’ is in use, try the next one.
If we have a hash table with 11 buckets and the silly hash function $hash(x) = x$, what will the hash table look like after inserting 3, 43, 8, 11, 14, 25, 23, 44 using open addressing?

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bucket = hash(x) % 11

If 'bucket' is in use, try the next one
```

<table>
<thead>
<tr>
<th>Bucket</th>
<th>Value</th>
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<tbody>
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If we have a hash table with 11 buckets and the silly hash function $hash(x) = x$, what will the hash table look like after inserting 3, 43, 8, 11, 14, 25, 23, 44 using **open addressing**?

$$bucket = hash(x) \mod 11$$

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If we have a hash table with 11 buckets and the silly hash function $\text{hash}(x) = x$, what will the hash table look like after inserting 3, 43, 8, 11, 14, 25, 23, 44 using open addressing?

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<th>Bucket</th>
<th>Values</th>
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<td>43</td>
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</tbody>
</table>

bucket = hash(x) % 11

If ‘bucket’ is in use, try the next one.
If we have a hash table with 11 buckets and the silly hash function $\text{hash}(x) = x$, what will the hash table look like after inserting 3, 43, 8, 11, 14, 25, 23, 44 using open addressing?

bucket = hash(x) % 11

If ‘bucket’ is in use, try the next one

<table>
<thead>
<tr>
<th>0</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>23</td>
</tr>
<tr>
<td>2</td>
<td>44</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>43</td>
</tr>
</tbody>
</table>
If we have a hash table with 11 buckets and the silly hash function $hash(x) = x$, what will the hash table look like after inserting 3, 43, 8, 11, 14, 25, 23, 44 using **buckets + chaining**?
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<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
If we have a hash table with 11 buckets and the silly hash function $hash(x) = x$, what will the hash table look like after inserting 3, 43, 8, 11, 14, 25, 23, 44 using **buckets + chaining**?

\[
\text{bucket} = \text{hash}(x) \% 11
\]
If we have a hash table with 11 buckets and the silly hash function \( \text{hash}(x) = x \), what will the hash table look like after inserting 3, 43, 8, 11, 14, 25, 23, 44 using \textbf{buckets + chaining}?

\[
\text{bucket} = \text{hash}(x) \mod 11
\]

If ‘bucket’ is in use, add the item to the chain.
If we have a hash table with 11 buckets and the silly hash function $hash(x) = x$, what will the hash table look like after inserting 3, 43, 8, 11, 14, 25, 23, 44 using **buckets + chaining**?

\[
\begin{array}{c|c}
\text{bucket} & \text{hash(x) \% 11} \\
\hline
0 & 0 \\
1 & 1 \\
2 & 2 \\
3 & 3 \\
4 & 4 \\
5 & 5 \\
6 & 6 \\
7 & 7 \\
8 & 8 \\
9 & 9 \\
10 & 10 \\
\end{array}
\]

If ‘bucket’ is in use, add the item to the chain
If we have a hash table with 11 buckets and the silly hash function \( hash(x) = x \), what will the hash table look like after inserting 3, 43, 8, 11, 14, 25, 23, 44 using buckets + chaining?

\[ \text{bucket} = \text{hash}(x) \mod 11 \]

If ‘bucket’ is in use, add the item to the chain

- Bucket 0
  - Item 3
- Bucket 1
  - Item 43
- Bucket 2
  - Item 8
- Bucket 3
  - Item 11
- Bucket 4
  - Item 14
- Bucket 5
- Bucket 6
- Bucket 7
- Bucket 8
- Bucket 9
- Bucket 10
  - Item 25
  - Item 23
  - Item 44
If we have a hash table with 11 buckets and the silly hash function $hash(x) = x$, what will the hash table look like after inserting 3, 43, 8, 11, 14, 25, 23, 44 using **buckets + chaining**?

\[
\begin{align*}
\text{bucket} &= hash(x) \mod 11 \\
\text{If ‘bucket’ is in use, add the item to the chain}
\end{align*}
\]

- Bucket 0: 3
- Bucket 1: 43
- Bucket 3: 8
- Bucket 8: 43
If we have a hash table with 11 buckets and the silly hash function \( hash(x) = x \), what will the hash table look like after inserting 3, 43, 8, 11, 14, 25, 23, 44 using **buckets + chaining**?

\[
\begin{align*}
0 & \quad 11 \\
1 & \\
2 & \\
3 & 3 \\
4 & \\
5 & \\
6 & \\
7 & \\
8 & 8 \\
9 & \\
10 & 43
\end{align*}
\]

bucket = \( hash(x) \) % 11

If ‘bucket’ is in use, add the item to the chain
If we have a hash table with 11 buckets and the silly hash function $hash(x) = x$, what will the hash table look like after inserting 3, 43, 8, 11, 14, 25, 23, 44 using **buckets + chaining**?

bucket = hash(x) % 11

If ‘bucket’ is in use, add the item to the chain

```
0  11
1
2
3  3  14
4
5
6
7
8  8
9
10  43
```
If we have a hash table with 11 buckets and the silly hash function \( \text{hash}(x) = x \), what will the hash table look like after inserting 3, 43, 8, 11, 14, 25, 23, 44 using **buckets + chaining**?

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bucket = hash(x) % 11

If ‘bucket’ is in use, add the item to the chain

<table>
<thead>
<tr>
<th>Bucket</th>
<th>Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>11</td>
</tr>
<tr>
<td>1</td>
<td>23</td>
</tr>
<tr>
<td>3</td>
<td>3, 14, 25</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
</tr>
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<td>10</td>
<td>43</td>
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\[
\begin{array}{c|c|c}
\text{bucket} & \text{item} & \text{item} \\
0 & 11 & 44 \\
1 & 23 & \\
2 & 3 & 14 & 25 \\
4 & 8 & \\
6 & 43 & \\
\end{array}
\]

\text{bucket} = \text{hash}(x) \% 11

If ‘bucket’ is in use, add the item to the chain.
If we have a hash table with 11 buckets and the silly hash function \( \text{hash}(x) = x \), what will the hash table look like after inserting 3, 43, 8, 11, 14, 25, 23, 44 using **buckets + chaining**?

\[
\begin{array}{c|c|c}
0 & 11 & 44 \\
1 & 23 & \\
2 & 3 & 14 & 25 \\
3 & 8 & \\
4 & 43 & \\
5 & 6 & 7 & 8 & 9 & 10 \\
\end{array}
\]

If ‘bucket’ is in use, add the item to the chain

What is the **table load**?
If we have a hash table with 11 buckets and the silly hash function $hash(x) = x$, what will the hash table look like after inserting 3, 43, 8, 11, 14, 25, 23, 44 using buckets + chaining?

```
0
1
2
3
4
5
6
7
8
9
10
```

<table>
<thead>
<tr>
<th>bucket</th>
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<td>8</td>
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<td></td>
</tr>
</tbody>
</table>

bucket = hash(x) % 11

If ‘bucket’ is in use, add the item to the chain

What is the **table load**?

8 items / 11 buckets
Does every key in a hash table need to be unique?
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Yes, that’s the point of storing key/value pairs
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Does each key in a hash table need to hash to a unique value?
Does every key in a hash table need to be unique?

Yes, that’s the point of storing key/value pairs.

Does each key in a hash table need to hash to a unique value?

No, but we should use a hash function with as few collisions as possible.
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Yes, that’s the point of storing key/value pairs

Does each key in a hash table need to hash to a unique value? 
No, but we should use a hash function with as few collisions as possible

Does hash table performance increase or decrease as the number of buckets increases?
Does every key in a hash table need to be unique?
Yes, that’s the point of storing key/value pairs

Does each key in a hash table need to hash to a unique value?
No, but we should use a hash function with as few collisions as possible

Does hash table performance increase or decrease as the number of buckets increases?
It should increase
Simulate **depth-first search** on this graph.
Simulate **depth-first search** on this graph

Store vertices in a **stack** (last in, first out)
Simulate **depth-first search** on this graph

Store vertices in a **stack**
(last in, first out)

Add nodes in counter-clockwise order
Simulate **depth-first search** on this graph

Store vertices in a **stack** (last in, first out)

Reachable:

Known:
Simulate **depth-first search** on this graph

- **Known:** A

- **Reachable:** A

- **Stack:** (last in, first out)

- **Add nodes in counter-clockwise order**
Simulate **depth-first search** on this graph

Store vertices in a *stack* (last in, first out)

Add nodes in counter-clockwise order

**Reachable:** A

**Known:**
Simulate **depth-first search** on this graph

- **Reachable:** A
- **Known:** B

Store vertices in a **stack** (last in, first out)

Add nodes in counter-clockwise order
Simulate **depth-first search** on this graph

Store vertices in a *stack* (last in, first out)

Add nodes in counter-clockwise order

Reachable: A, B

Known:
Simulate **depth-first search** on this graph

**Store vertices in a stack** (last in, first out)

**Known:**
- C

**Reachable:**
- A
- B

**Add nodes in counter-clockwise order**
Simulate **depth-first search** on this graph

**Reachable:** A, B

**Known:** C, E

Store vertices in a **stack** (last in, first out)

Add nodes in counter-clockwise order
Simulate **depth-first search** on this graph

- **Known:** A, B, C, E, D
- **Reachable:** A, B
- **Add nodes in counter-clockwise order**
- **Store vertices in a stack** (last in, first out)
Simulate **depth-first search** on this graph

**Reachable:** A, B, D

**Known:** C, E

Store vertices in a **stack** (last in, first out)

Add nodes in counter-clockwise order
Simulate **depth-first search** on this graph

Store vertices in a **stack** (last in, first out)

Add nodes in counter-clockwise order

Reachable: \( \{A, B, D\} \)

Known: \( \{C, E, F\} \)
Simulate **depth-first search** on this graph

Store vertices in a *stack* (last in, first out)

Add nodes in counter-clockwise order

Reachable: A B D F

Known: C E

Store vertices in a *stack* (last in, first out)
Simulate **depth-first search** on this graph

Store vertices in a *stack* (last in, first out)

Reachable: A B D F E

Known: C

Add nodes in counter-clockwise order
Simulate **depth-first search** on this graph

- Store vertices in a **stack** (last in, first out)
- Add nodes in counter-clockwise order

Reachable: A B D F E C

Known: A B D F E C
Simulate **breadth-first search** on this graph.
Simulate **breadth-first search** on this graph

Store vertices in a **queue**
(first in, first out)
Simulate **breadth-first search** on this graph

- Store vertices in a **queue** (first in, first out)
- Add nodes in counter-clockwise order
Simulate **breadth-first search** on this graph.

- Store vertices in a **queue** (first in, first out).
- Add nodes in counter-clockwise order.

Reachable: 

Known:
Simulate **breadth-first search** on this graph

Store vertices in a **queue** (first in, first out)

Add nodes in counter-clockwise order

** reachable:**

** known:** A
Simulate **breadth-first search** on this graph

Store vertices in a *queue* (first in, first out)

Add nodes in counter-clockwise order

Reachable:  

Known:
Simulate **breadth-first search** on this graph

Store vertices in a *queue* (first in, first out)

Reachable: A

Known: B

Add nodes in counter-clockwise order
Simulate **breadth-first search** on this graph

Store vertices in a *queue* (first in, first out)

Add nodes in counter-clockwise order

Reachable:  A  B

Known:
Simulate **breadth-first search** on this graph

Store vertices in a **queue** (first in, first out)

Add nodes in counter-clockwise order

Reachable: A, B

Known: C
Simulate **breadth-first search** on this graph

Store vertices in a *queue* (first in, first out)

Add nodes in counter-clockwise order

Reachable: \( A, B, D, F \)

Known: \( C, E \)
Simulate **breadth-first search** on this graph.

- Store vertices in a **queue** (first in, first out)
- Add nodes in counter-clockwise order

**Reachable:** A, B

**Known:** C, E, D
Simulate **breadth-first search** on this graph

- **Known:** A, B, C, D, E
- **Reachable:** A, B, C

Store vertices in a **queue** (first in, first out)

Add nodes in counter-clockwise order
Simulate **breadth-first search** on this graph

Store vertices in a **queue** (first in, first out)

Add nodes in counter-clockwise order

Reachable: \( \{A, B, C, E\} \)

Known: \( \{D\} \)
Simulate **breadth-first search** on this graph

- **Reachable:** A, B, C, D, E
- **Known:** A, B, C, D

*Store vertices in a *queue* (first in, first out)*

*Add nodes in counter-clockwise order*
Simulate **breadth-first search** on this graph

Store vertices in a *queue* (first in, first out)

Add nodes in counter-clockwise order

Reachable: \( A, B, C, E, D \)

Known: \( D \)
Simulate **breadth-first search** on this graph

Store vertices in a **queue** (first in, first out)

Add nodes in counter-clockwise order

Reachable: A B C E D F

Known: D F
Simulate **breadth-first search** on this graph

Store vertices in a **queue** (first in, first out)

Add nodes in counter-clockwise order

**Reachable:** A B C D E F

**Known:** F
Simulate **breadth-first search** on this graph

Store vertices in a **queue** (first in, first out)

Add nodes in counter-clockwise order

Reachable: A B C E D F

Known: