

CS 161

Intro to CS I

Recursion

Recursion

- What is it?
 - Function that calls itself 1 or more times (directly or indirectly)
 - Has 1 or more base case for stopping
 - Inductive reasoning: general case must eventually be reduced to a base case

Example: Drawing Rectangles

- Iterative Solution:

```
void draw_rect(int i) {  
    for( ; i > 0; i--){  
        cout << "*****" << endl;  
        cout << " *      *" << endl;  
        cout << "*****" << endl << endl;  
    }  
}
```

Example: Drawing Rectangles

- Recursive Solution

```
void draw_rect(int i) {  
    if(i>0){    //Base case  
        draw_rect(--i); //Recursive call  
        cout << "*****" << endl;  
        cout << " *      *" << endl;  
        cout << "*****" << endl << endl;  
    }  
}
```

What is different when we call after?

- Recursive Solution

```
void draw_rect(int i) {  
    if(i>0){    //Base case  
        cout << "*****" << endl;  
        cout << " *      *" << endl;  
        cout << "*****" << endl << endl;  
        draw_rect(--i);    //Recursive call  
    }  
}
```

Example: Factorial

- Definition

$$0! = 1;$$

$$n! = n * (n-1) * \dots * (n-(n-1)) * 1 = n * (n-1)! ; n > 0$$

Iterative Factorial

factorial(0) = 1;

factorial(n) = $n * n-1 * n-2 * \dots * n-(n-1) * 1$;

```
long factorial(int n) {  
    long fact;  
    if(n==0)  
        fact=1;  
    else  
        for(fact=n; n > 1; n--)  
            fact=fact*(n-1);  
    return fact;  
}
```

Recursive Factorial

factorial(0) = 1;

factorial(n) = n*factorial(n-1);

```
long factorial(int n) {  
    if (n == 0)    // Base case  
        return 1;  
    else  
        return n * factorial(n - 1);    // Recursive call  
}
```


Computing Factorial Iteratively

factorial(4)

```
factorial(0) = 1;  
factorial(n) = n*(n-1)*...*2*1;
```

Computing Factorial Iteratively

$$\text{factorial}(4) = 4 * 3$$

```
factorial(0) = 1;  
factorial(n) = n*(n-1)*...*2*1;
```

Computing Factorial Iteratively

$$\begin{aligned}\text{factorial}(4) &= 4 * 3 \\ &= 12 * 2\end{aligned}$$

```
factorial(0) = 1;  
factorial(n) = n*(n-1)*...*2*1;
```

Computing Factorial Iteratively

$$\begin{aligned}\text{factorial}(4) &= 4 * 3 \\ &= 12 * 2 \\ &= 24 * 1\end{aligned}$$

```
factorial(0) = 1;  
factorial(n) = n*(n-1)*...*2*1;
```

Computing Factorial Iteratively

$$\begin{aligned}\text{factorial}(4) &= 4 * 3 \\ &= 12 * 2 \\ &= 24 * 1 \\ &= 24\end{aligned}$$

```
factorial(0) = 1;  
factorial(n) = n*(n-1)*...*2*1;
```

Computing Factorial Recursively

factorial(4)

```
factorial(0) = 1;  
factorial(n) = n*factorial(n-1);
```

Computing Factorial Recursively

$$\text{factorial}(4) = 4 * \text{factorial}(3)$$

```
factorial(0) = 1;  
factorial(n) = n*factorial(n-1);
```

Computing Factorial Recursively

```
factorial(0) = 1;  
factorial(n) = n * factorial(n-1);
```

$$\begin{aligned}\text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * (3 * \text{factorial}(2))\end{aligned}$$

Computing Factorial Recursively

```
factorial(0) = 1;  
factorial(n) = n * factorial(n-1);
```

$$\begin{aligned}\text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * (3 * \text{factorial}(2)) \\ &= 4 * (3 * (2 * \text{factorial}(1)))\end{aligned}$$

Computing Factorial Recursively

```
factorial(0) = 1;  
factorial(n) = n * factorial(n-1);
```

$$\begin{aligned} \text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * (3 * \text{factorial}(2)) \\ &= 4 * (3 * (2 * \text{factorial}(1))) \\ &= 4 * (3 * (2 * (1 * \text{factorial}(0)))) \end{aligned}$$

Computing Factorial Recursively

```
factorial(0) = 1;  
factorial(n) = n * factorial(n-1);
```

$$\begin{aligned} \text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * (3 * \text{factorial}(2)) \\ &= 4 * (3 * (2 * \text{factorial}(1))) \\ &= 4 * (3 * (2 * (1 * \text{factorial}(0)))) \\ &= 4 * (3 * (2 * (1 * 1))) \end{aligned}$$

Computing Factorial Recursively

```
factorial(0) = 1;  
factorial(n) = n * factorial(n-1);
```

$$\begin{aligned} \text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * (3 * \text{factorial}(2)) \\ &= 4 * (3 * (2 * \text{factorial}(1))) \\ &= 4 * (3 * (2 * (1 * \text{factorial}(0)))) \\ &= 4 * (3 * (2 * (1 * 1))) \\ &= 4 * (3 * (2 * 1)) \end{aligned}$$

Computing Factorial Recursively

```
factorial(0) = 1;  
factorial(n) = n * factorial(n-1);
```

$$\begin{aligned} \text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * (3 * \text{factorial}(2)) \\ &= 4 * (3 * (2 * \text{factorial}(1))) \\ &= 4 * (3 * (2 * (1 * \text{factorial}(0)))) \\ &= 4 * (3 * (2 * (1 * 1))) \\ &= 4 * (3 * (2 * 1)) \\ &= 4 * (3 * 2) \end{aligned}$$

Computing Factorial Recursively

```
factorial(0) = 1;  
factorial(n) = n * factorial(n-1);
```

$$\begin{aligned} \text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * (3 * \text{factorial}(2)) \\ &= 4 * (3 * (2 * \text{factorial}(1))) \\ &= 4 * (3 * (2 * (1 * \text{factorial}(0)))) \\ &= 4 * (3 * (2 * (1 * 1))) \\ &= 4 * (3 * (2 * 1)) \\ &= 4 * (3 * 2) \\ &= 4 * 6 \end{aligned}$$

Computing Factorial Recursively

```
factorial(0) = 1;  
factorial(n) = n * factorial(n-1);
```

$$\begin{aligned}\text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * (3 * \text{factorial}(2)) \\ &= 4 * (3 * (2 * \text{factorial}(1))) \\ &= 4 * (3 * (2 * (1 * \text{factorial}(0)))) \\ &= 4 * (3 * (2 * (1 * 1))) \\ &= 4 * (3 * (2 * 1)) \\ &= 4 * (3 * 2) \\ &= 4 * 6 \\ &= 24\end{aligned}$$

Differences

- Pros
 - Readability
- Cons
 - Efficiency
 - Memory

Recursive Factorial

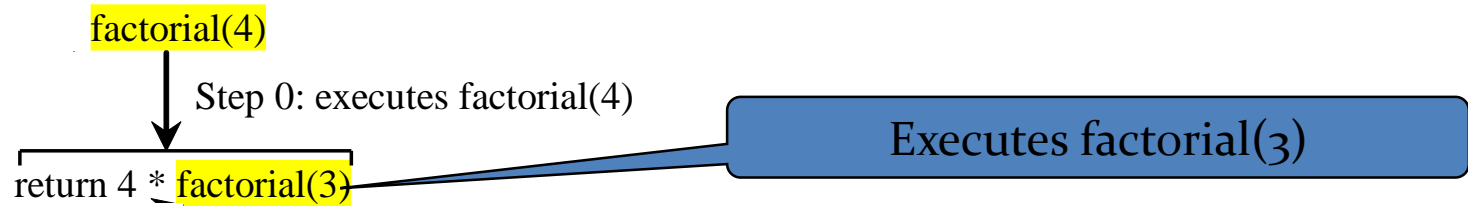
factorial(4)

Executes factorial(4)

Stack

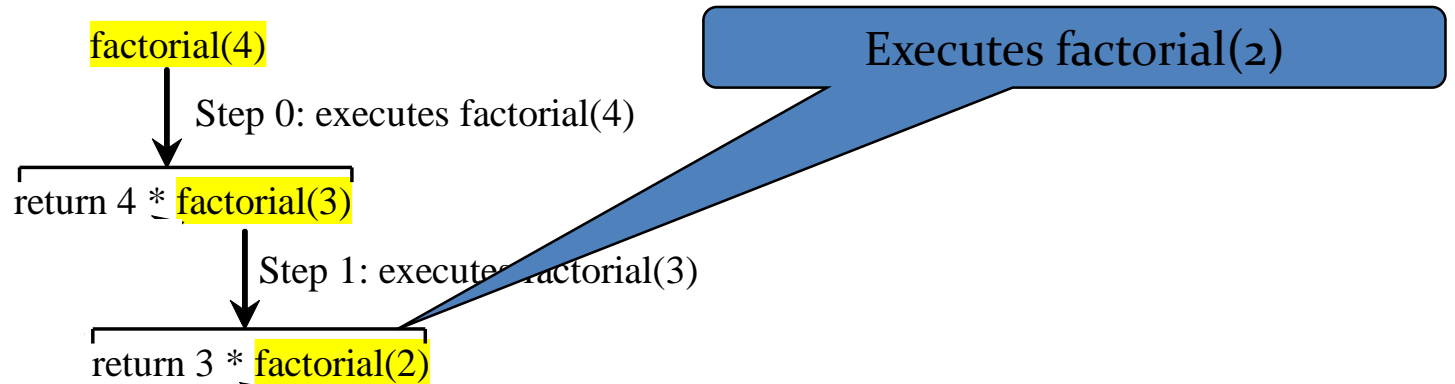
Main method

Recursive Factorial



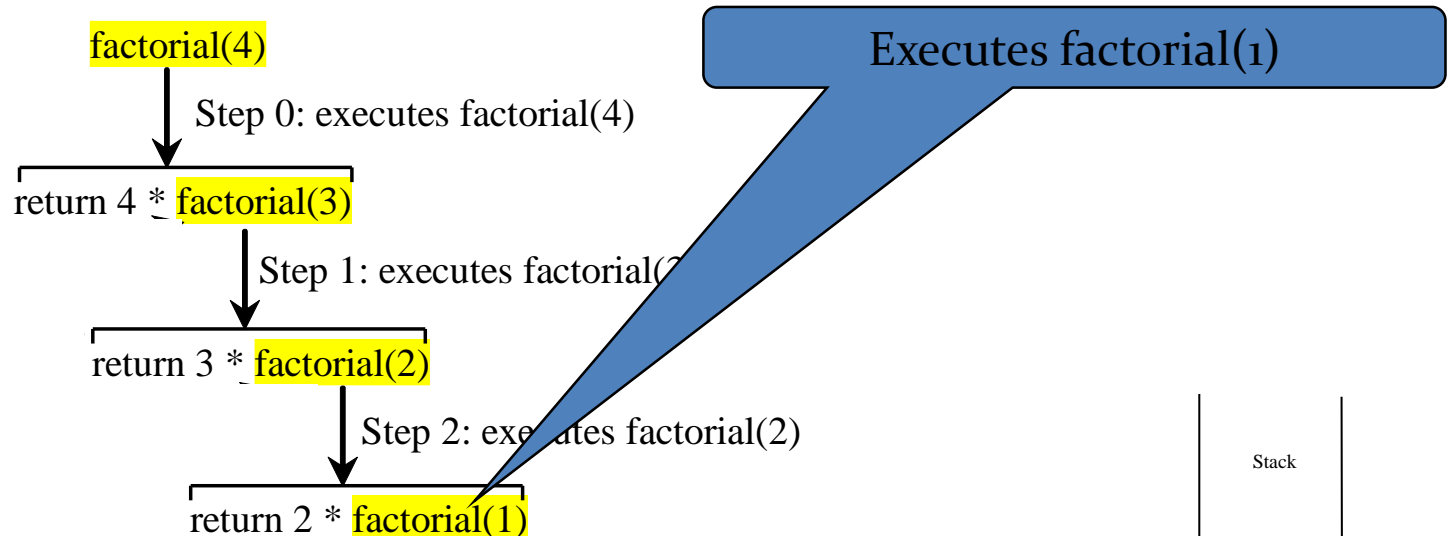
Stack
Space Required for factorial(4)
Main method

Recursive Factorial



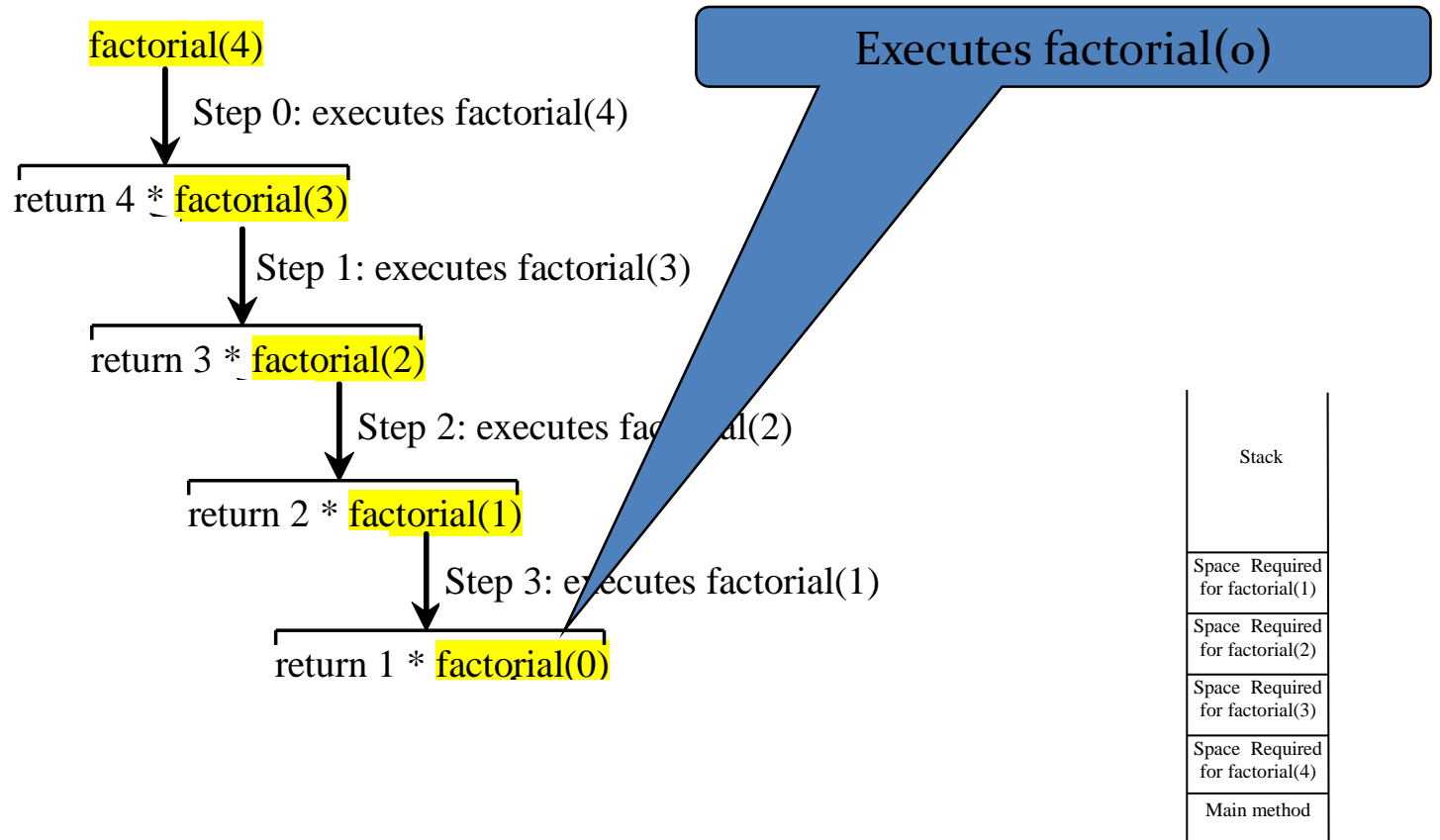
Stack
Space Required for factorial(3)
Space Required for factorial(4)
Main method

Recursive Factorial

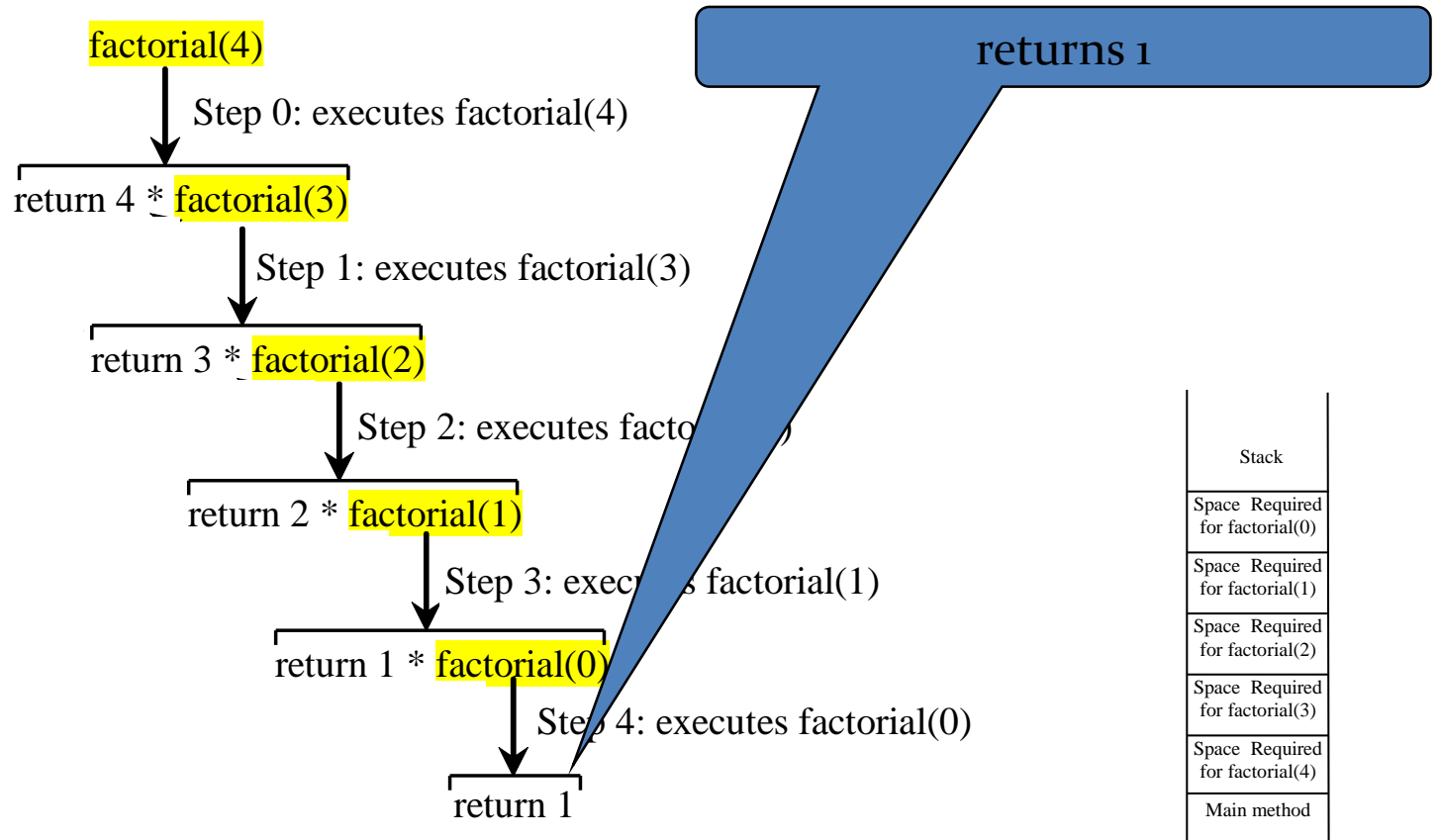


Stack
Space Required for factorial(2)
Space Required for factorial(3)
Space Required for factorial(4)
Main method

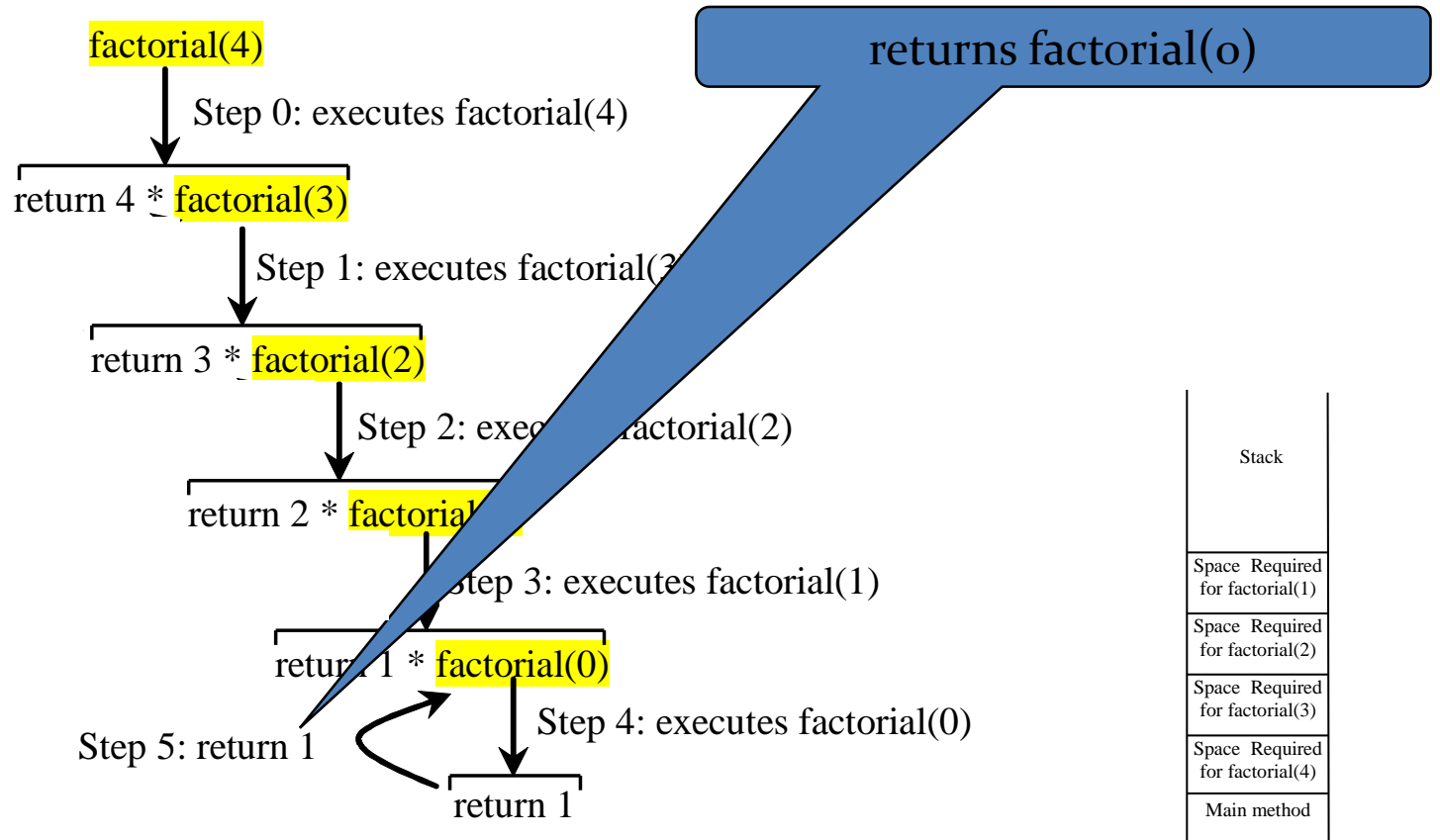
Recursive Factorial



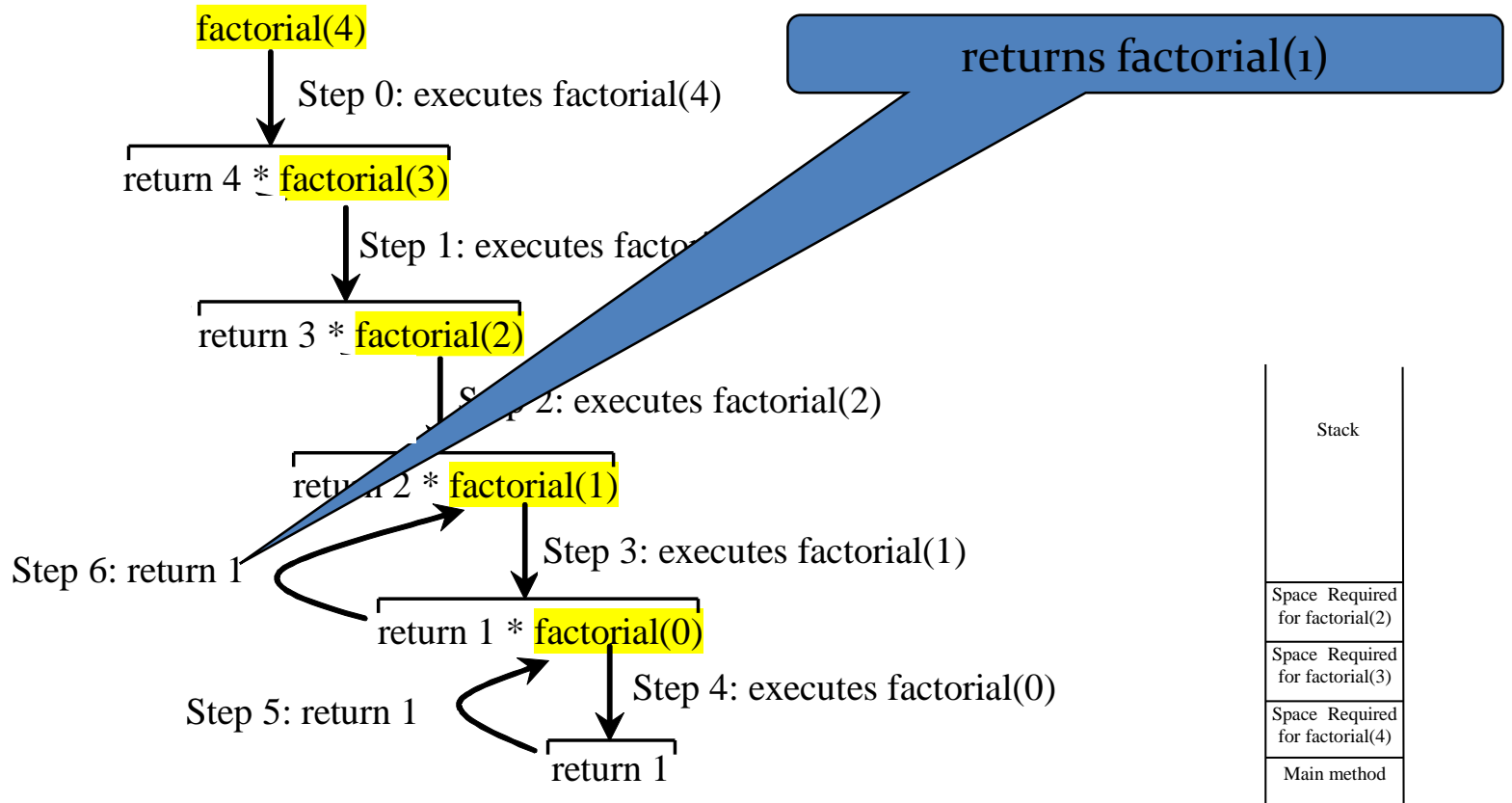
Recursive Factorial



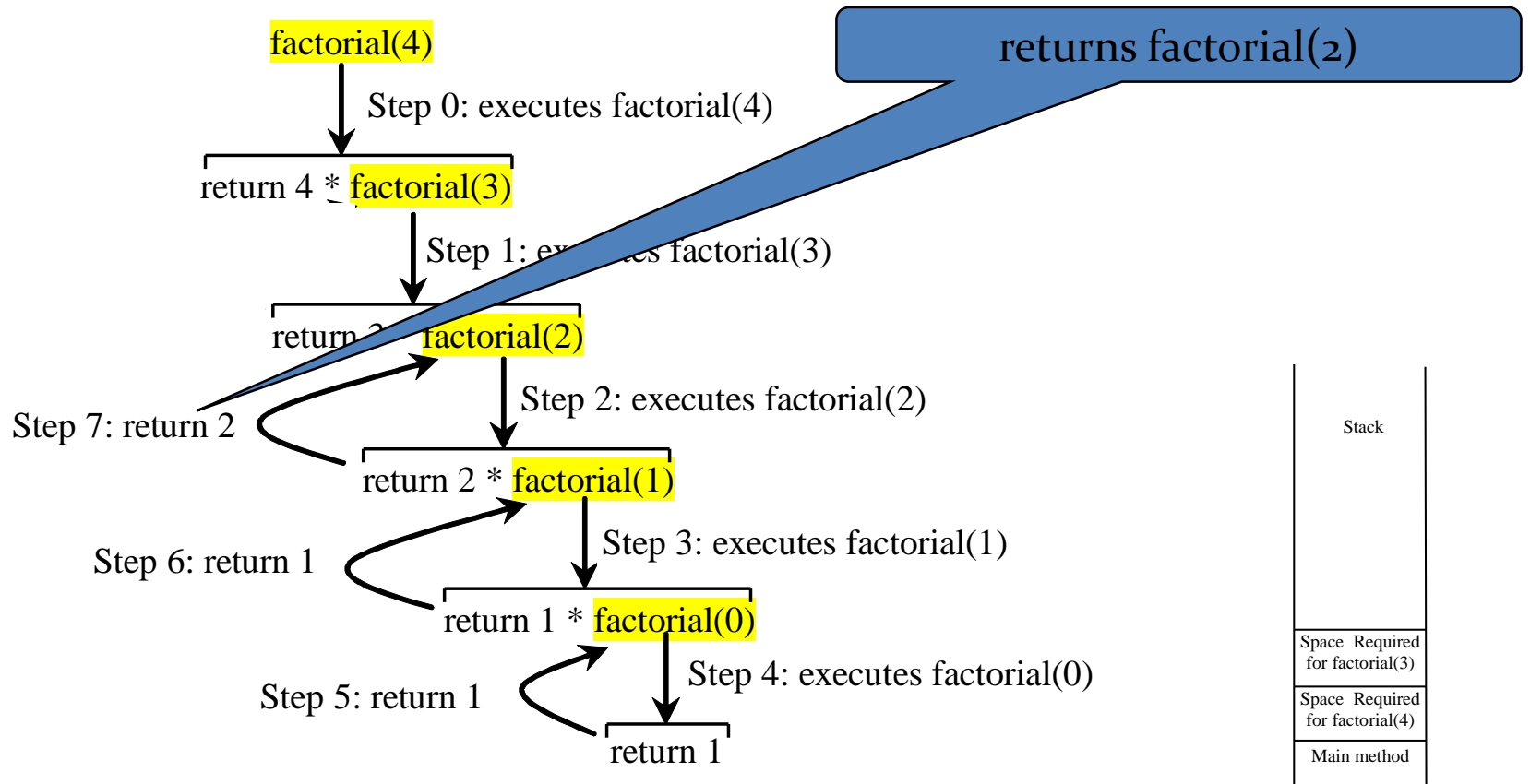
Recursive Factorial



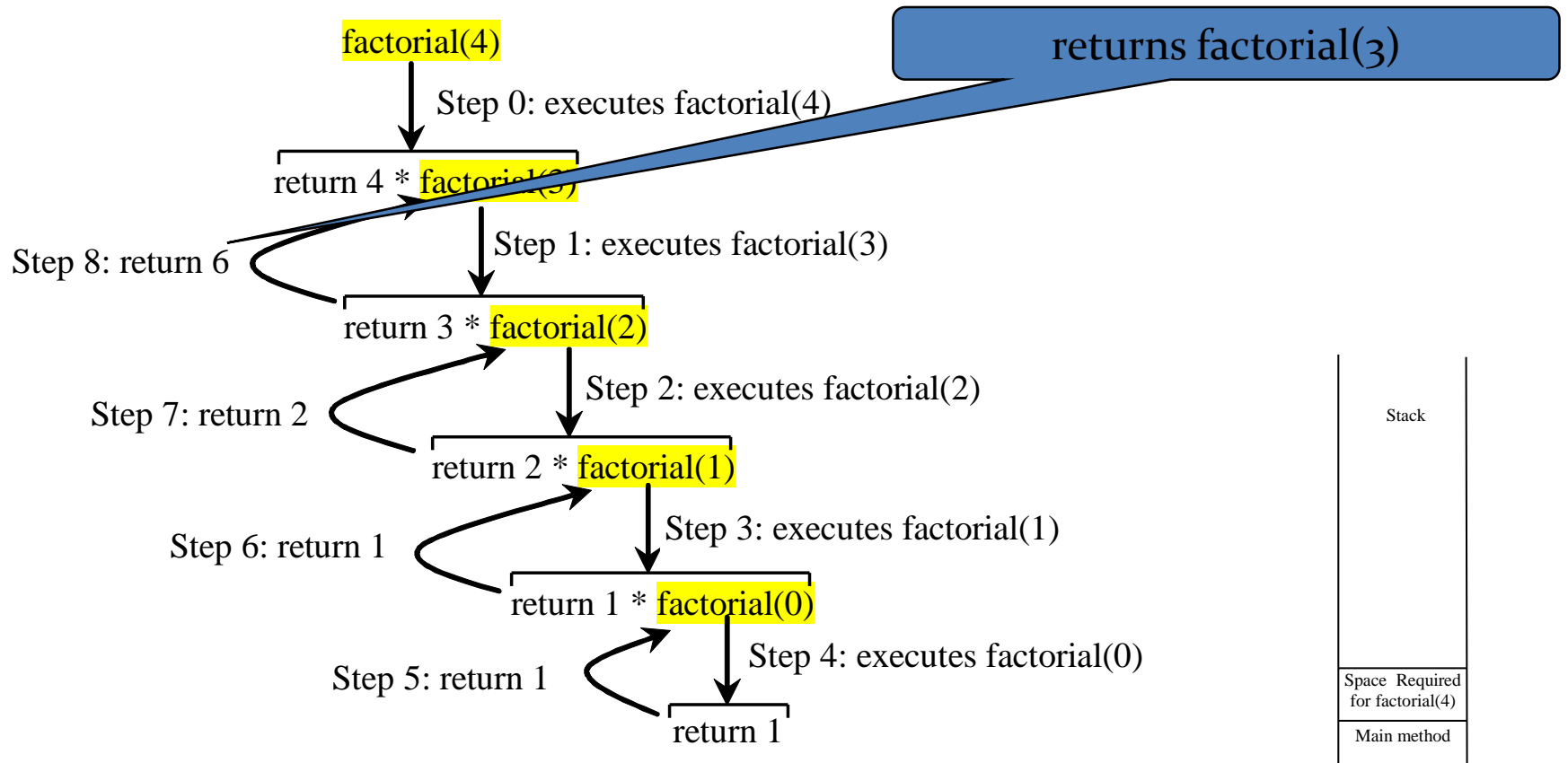
Recursive Factorial



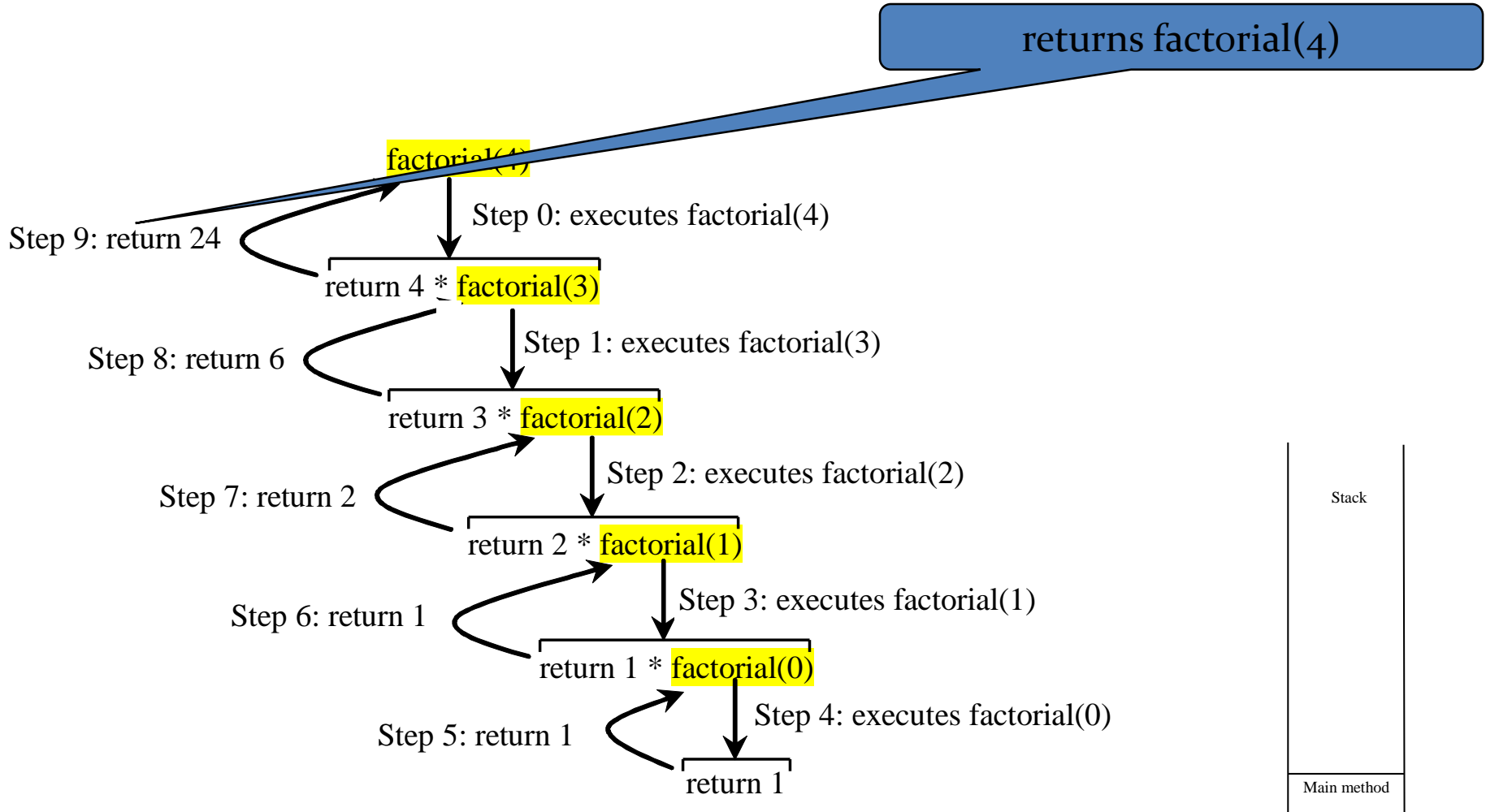
Recursive Factorial



Recursive Factorial



Recursive Factorial



In-class Exercise #4

- Get into groups of 4 – 5.
- Write your own recursive *int pwr()* function that takes two integers as arguments and returns the integer result.
 - What does the function prototype look like?
 - Now, write the function definition...