

CS 161

Intro to CS I

Recursion

Example: Factorial

- Definition

$$0! = 1;$$

$$n! = \underbrace{n * (n-1) * \dots * (n-(n-1)) * 1}_{\text{iterative}} = \underbrace{n * (n-1)!}_{\text{recursive}}; n > 0$$

Iterative Factorial

factorial(0) = 1;

factorial(n) = $n * n-1 * n-2 * \dots * n-(n-1) * 1$;

```
long factorial(int n) {  
    long fact;  
    if(n==0)  
        fact=1;  
    else  
        for(fact=n; n > 1; n--)  
            fact=fact*(n-1);  
    return fact;  
}
```

Recursive Factorial

```
factorial(0) = 1;  
factorial(n) = n * factorial(n-1);
```

```
long factorial(int n) {  
    if (n == 0)    // Base case  
        return 1;  
    else  
        return n * factorial(n - 1);    // Recursive call  
}
```

Computing Factorial Iteratively

factorial(4)

```
factorial(0) = 1;
```

```
factorial(n) = n*(n-1)*...*2*1;
```

Computing Factorial Iteratively

$$\text{factorial}(4) = 4 * 3$$

```
factorial(0) = 1;
```

```
factorial(n) = n*(n-1)*...*2*1;
```

Computing Factorial Iteratively

$$\begin{aligned} \text{factorial}(4) &= 4 * 3 \\ &= 12 * 2 \end{aligned}$$

```
factorial(0) = 1;
```


```
factorial(n) = n*(n-1)*...*2*1;
```

Computing Factorial Iteratively

$$\begin{aligned}\text{factorial}(4) &= 4 * 3 \\ &= 12 * 2 \\ &= 24 * 1\end{aligned}$$

```
factorial(0) = 1;  
factorial(n) = n*(n-1)*...*2*1;
```


Computing Factorial Iteratively

$$\begin{aligned}\text{factorial}(4) &= 4 * 3 \\ &= 12 * 2 \\ &= 24 * 1 \\ &= 24\end{aligned}$$


```
factorial(0) = 1;  
factorial(n) = n*(n-1)*...*2*1;
```

Computing Factorial Recursively

factorial(4)

```
factorial(0) = 1;  
factorial(n) = n*factorial(n-1);
```

Computing Factorial Recursively

$$\text{factorial}(4) = 4 * \underline{\text{factorial}(3)}$$

```
factorial(0) = 1;  
factorial(n) = n*factorial(n-1);
```

Computing Factorial Recursively

```
factorial(0) = 1;  
factorial(n) = n * factorial(n-1);
```

$$\begin{aligned} \text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * (3 * \underline{\text{factorial}(2)}) \end{aligned}$$

Computing Factorial Recursively

```
factorial(0) = 1;  
factorial(n) = n * factorial(n-1);
```

$$\begin{aligned}\text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * (3 * \text{factorial}(2)) \\ &= 4 * (3 * (2 * \text{factorial}(1)))\end{aligned}$$

Computing Factorial Recursively

```
factorial(0) = 1;  
factorial(n) = n * factorial(n-1);
```

$$\begin{aligned}\text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * (3 * \text{factorial}(2)) \\ &= 4 * (3 * (2 * \text{factorial}(1))) \\ &= 4 * (3 * (2 * (1 * \text{factorial}(0))))\end{aligned}$$

Computing Factorial Recursively

```
factorial(0) = 1;  
factorial(n) = n * factorial(n-1);
```

$$\begin{aligned} \text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * (3 * \text{factorial}(2)) \\ &= 4 * (3 * (2 * \text{factorial}(1))) \\ &= 4 * (3 * (2 * (1 * \text{factorial}(0)))) \\ &= 4 * (3 * (2 * (1 * 1))) \end{aligned}$$

Computing Factorial Recursively

```
factorial(0) = 1;
```

```
factorial(n) = n * factorial(n-1);
```

$$\begin{aligned} \text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * (3 * \text{factorial}(2)) \\ &= 4 * (3 * (2 * \text{factorial}(1))) \\ &= 4 * (3 * (2 * (1 * \text{factorial}(0)))) \\ &= 4 * (3 * (2 * (1 * 1))) \\ &= 4 * (3 * (2 * 1)) \end{aligned}$$


Computing Factorial Recursively

```
factorial(0) = 1;  
factorial(n) = n * factorial(n-1);
```

$$\begin{aligned} \text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * (3 * \text{factorial}(2)) \\ &= 4 * (3 * (2 * \text{factorial}(1))) \\ &= 4 * (3 * (2 * (1 * \text{factorial}(0)))) \\ &= 4 * (3 * (2 * (1 * 1))) \\ &= 4 * (3 * (2 * 1)) \\ &= 4 * (3 * 2) \end{aligned}$$

Computing Factorial Recursively

```
factorial(0) = 1;  
factorial(n) = n * factorial(n-1);
```

$$\begin{aligned} \text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * (3 * \text{factorial}(2)) \\ &= 4 * (3 * (2 * \text{factorial}(1))) \\ &= 4 * (3 * (2 * (1 * \text{factorial}(0)))) \\ &= 4 * (3 * (2 * (1 * 1))) \\ &= 4 * (3 * (2 * 1)) \\ &= 4 * (3 * 2) \\ &= 4 * 6 \end{aligned}$$


Computing Factorial Recursively

`factorial(0) = 1;`

`factorial(n) = n * factorial(n-1);`

$$\begin{aligned} \text{factorial}(4) &= 4 * \text{factorial}(3) \\ &= 4 * (3 * \text{factorial}(2)) \\ &= 4 * (3 * (2 * \text{factorial}(1))) \\ &= 4 * (3 * (2 * (1 * \text{factorial}(0)))) \\ &= 4 * (3 * (2 * (1 * 1))) \\ &= 4 * (3 * (2 * 1)) \\ &= 4 * (3 * 2) \\ &= 4 * 6 \\ &= 24 \end{aligned}$$

Differences

- Pros
 - Readability
- Cons
 - Efficiency
 - Memory

not a reason to avoid recursion

Recursive Factorial

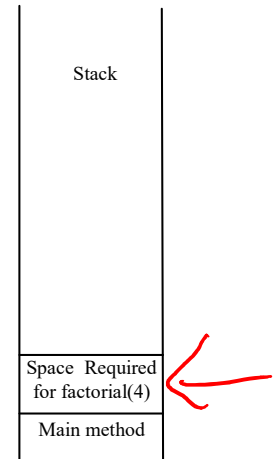
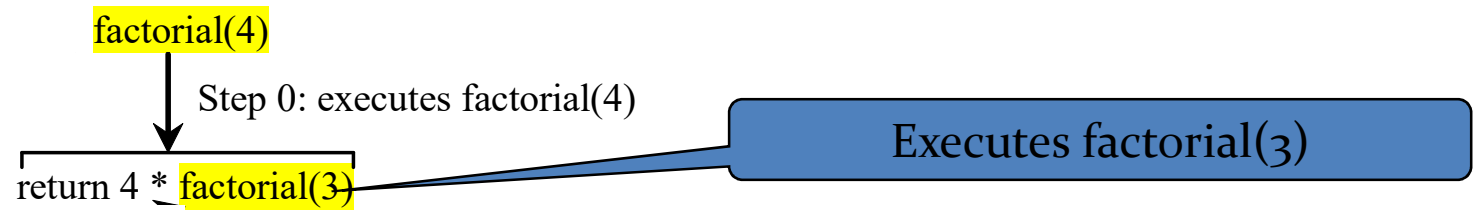
factorial(4)

Executes factorial(4)

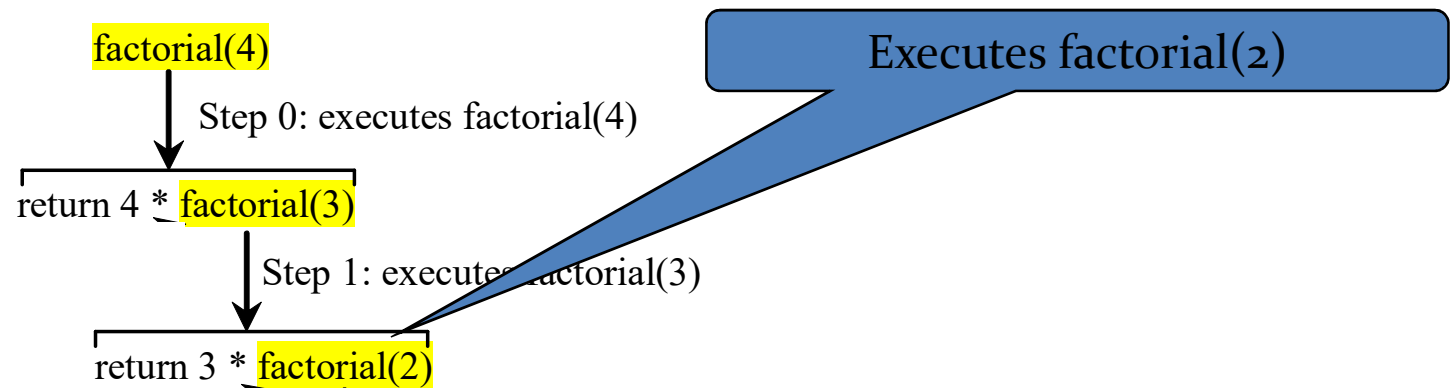
Stack

Main method

Recursive Factorial

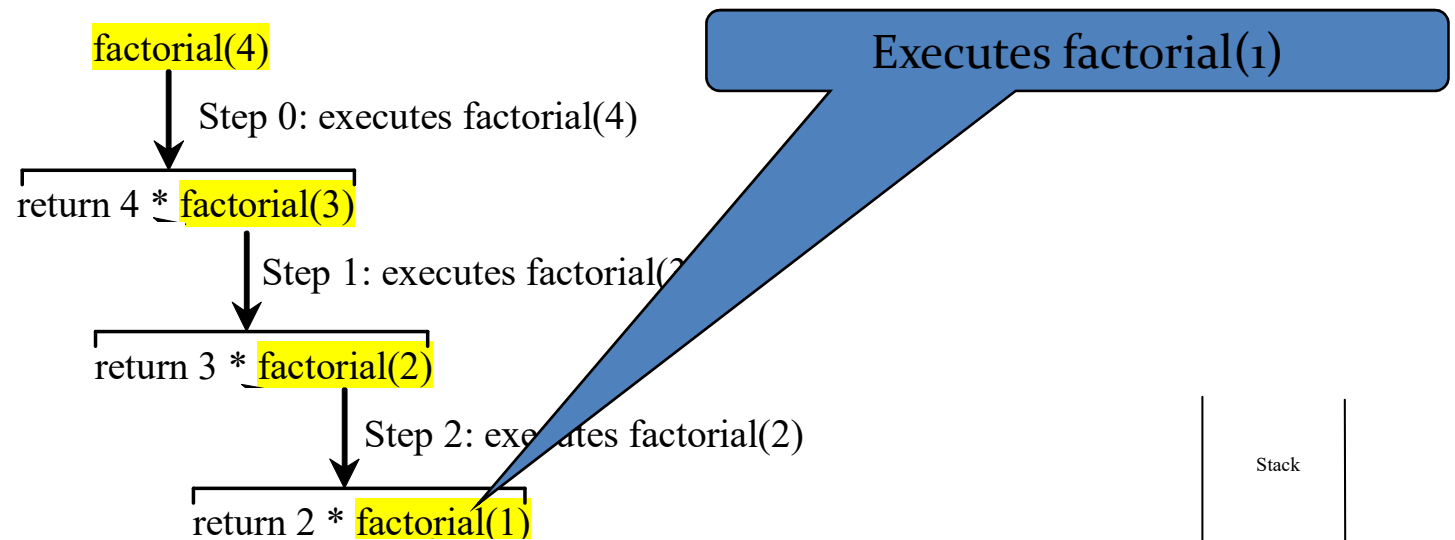


Recursive Factorial



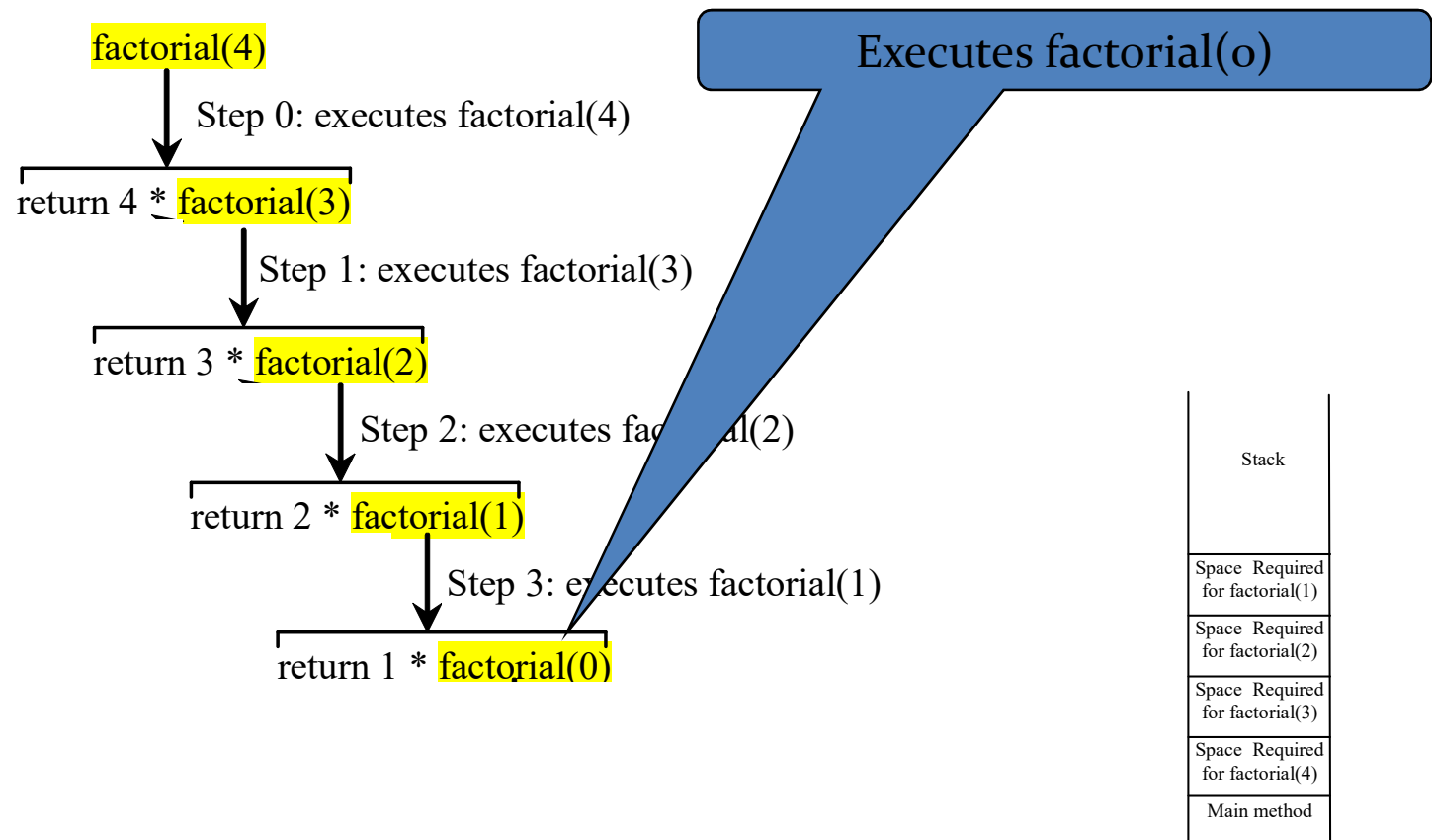
Stack
Space Required for factorial(3)
Space Required for factorial(4)
Main method

Recursive Factorial

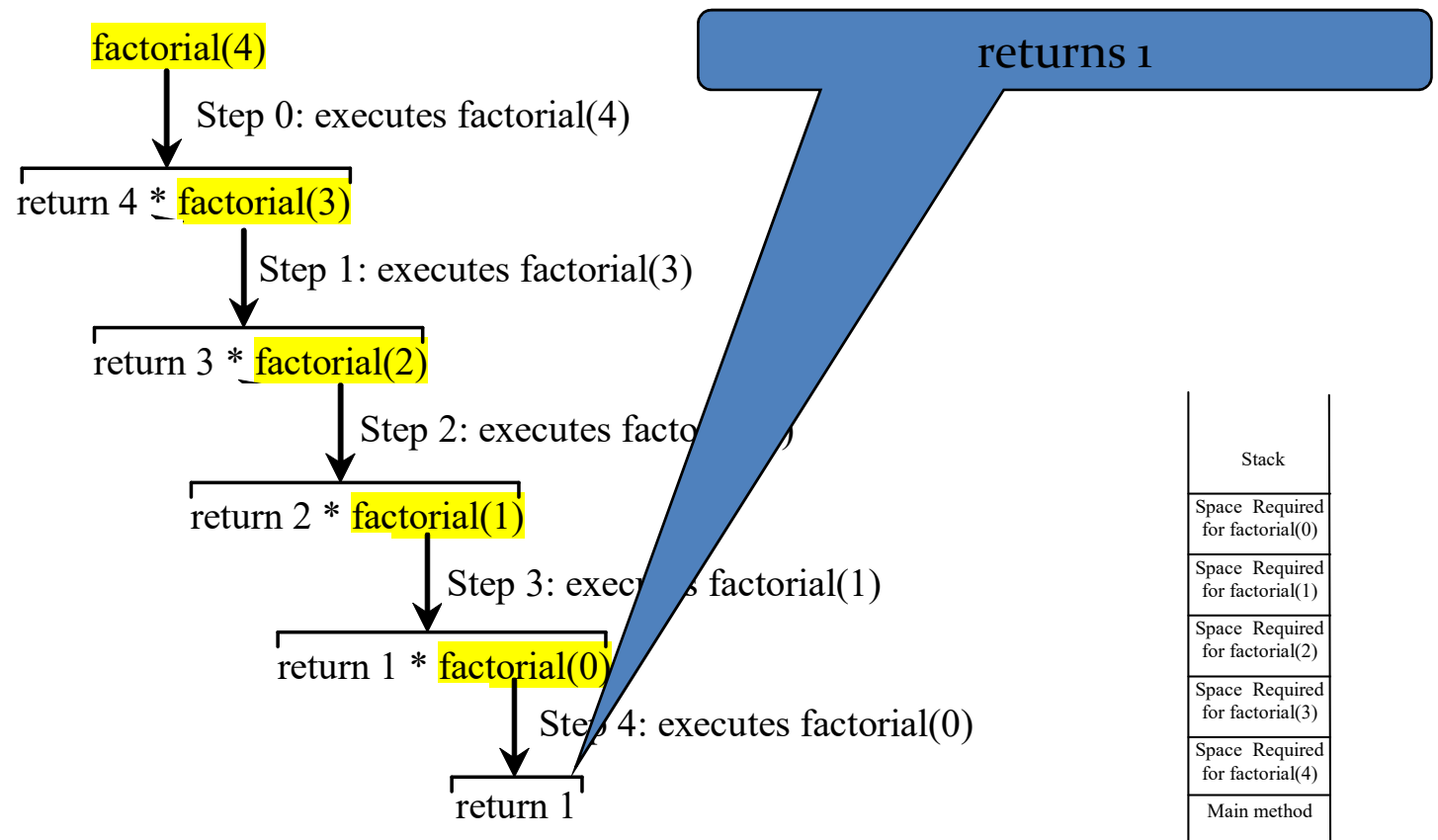


Stack
Space Required for factorial(2)
Space Required for factorial(3)
Space Required for factorial(4)
Main method

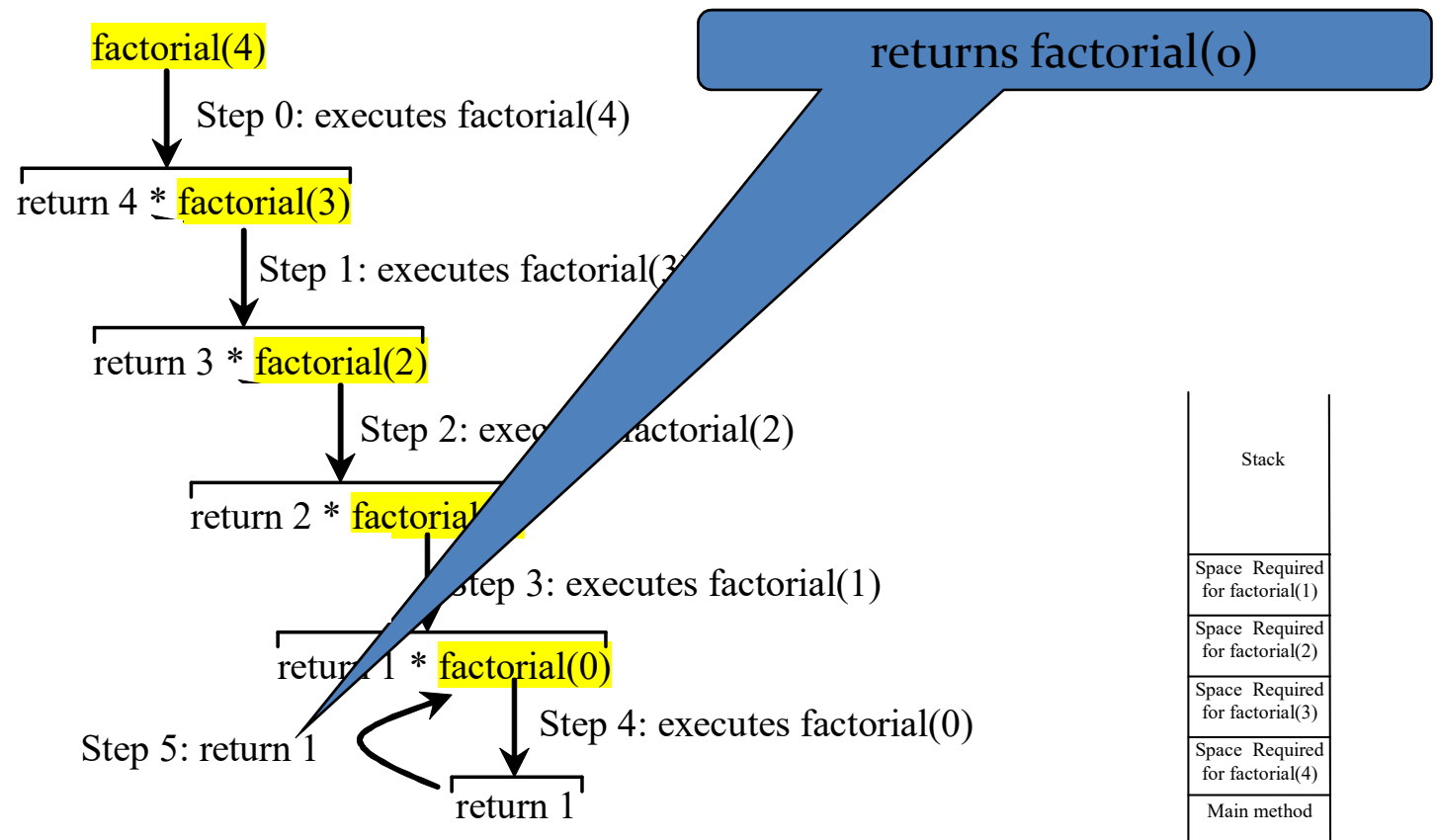
Recursive Factorial



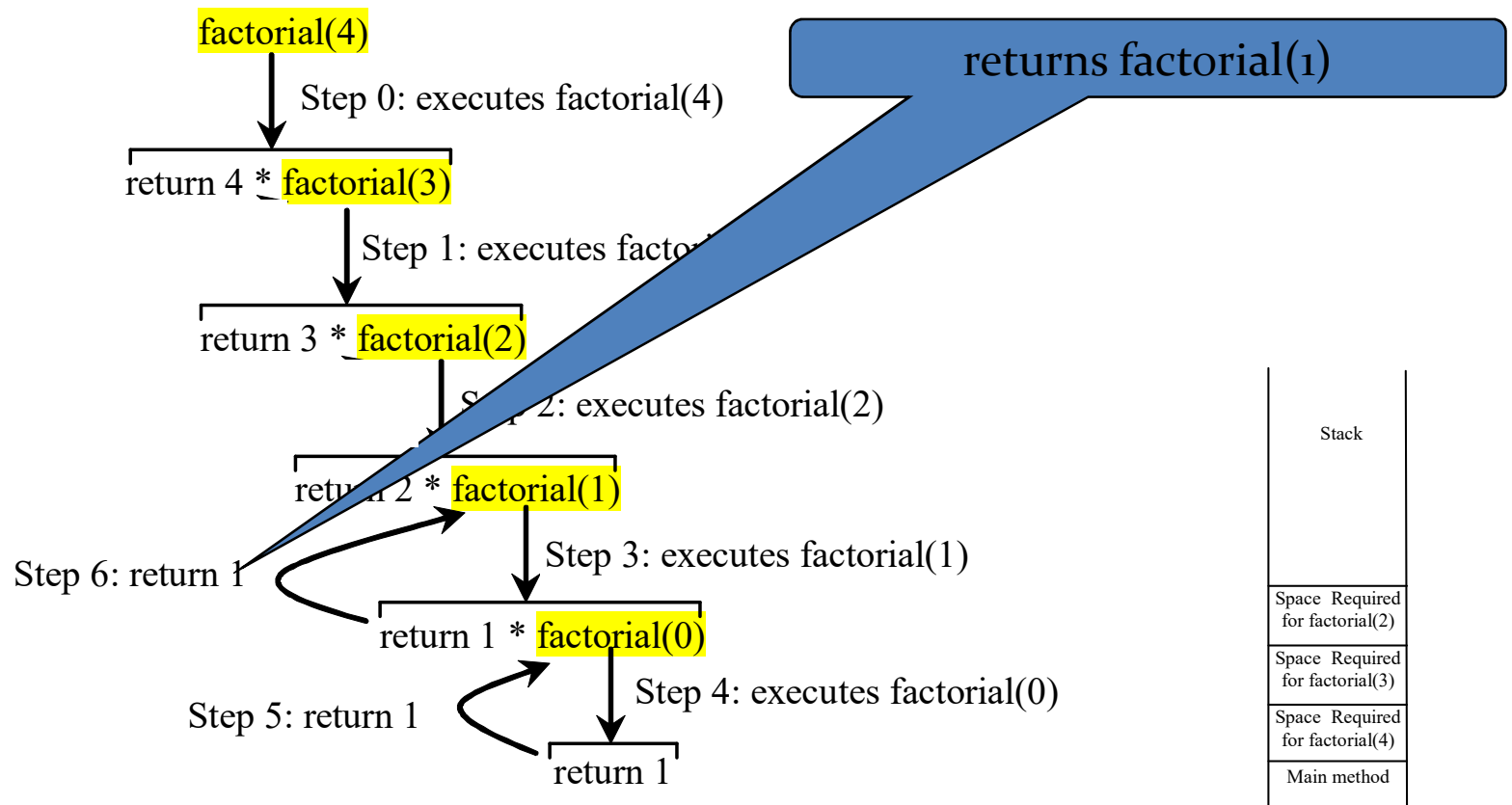
Recursive Factorial



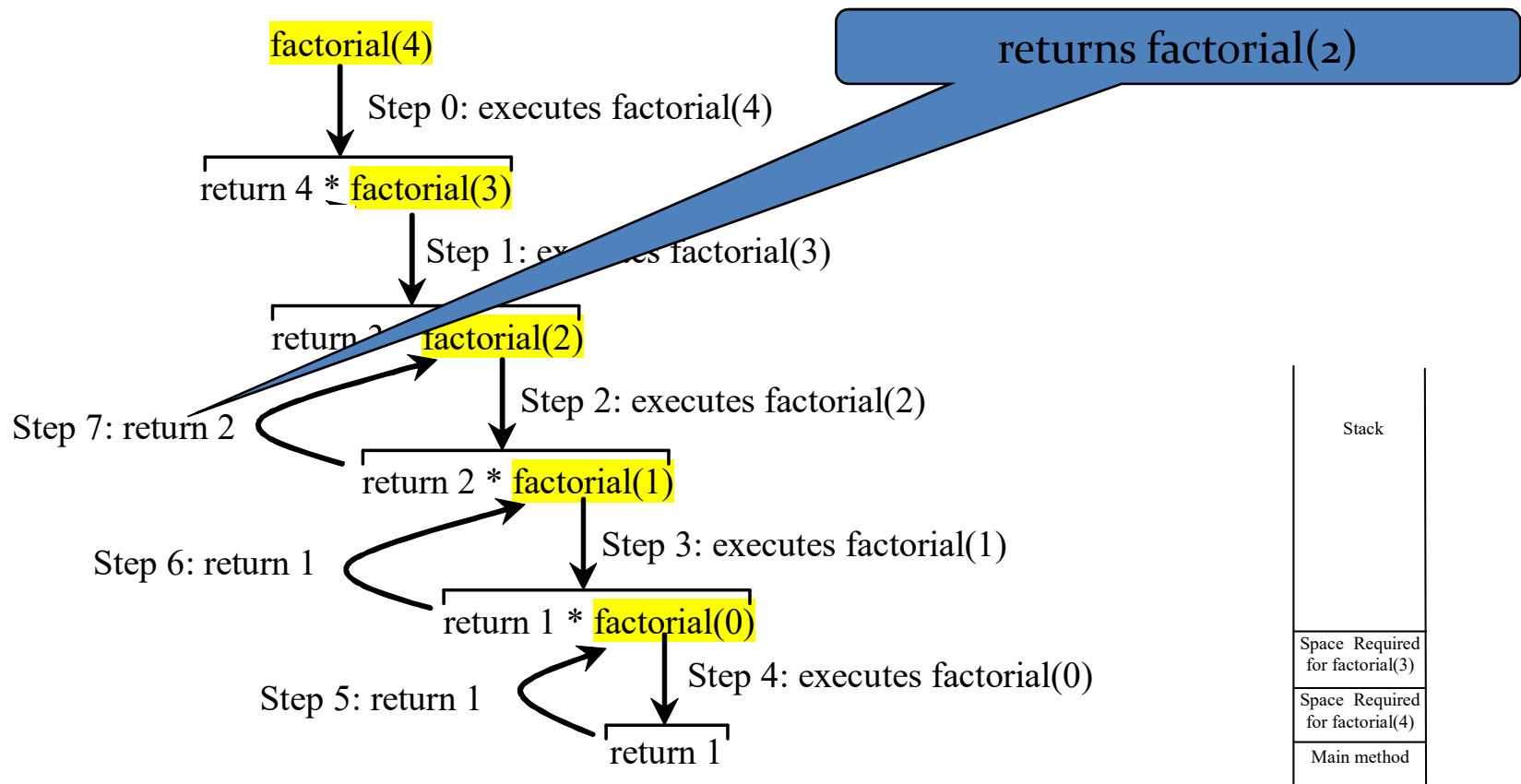
Recursive Factorial



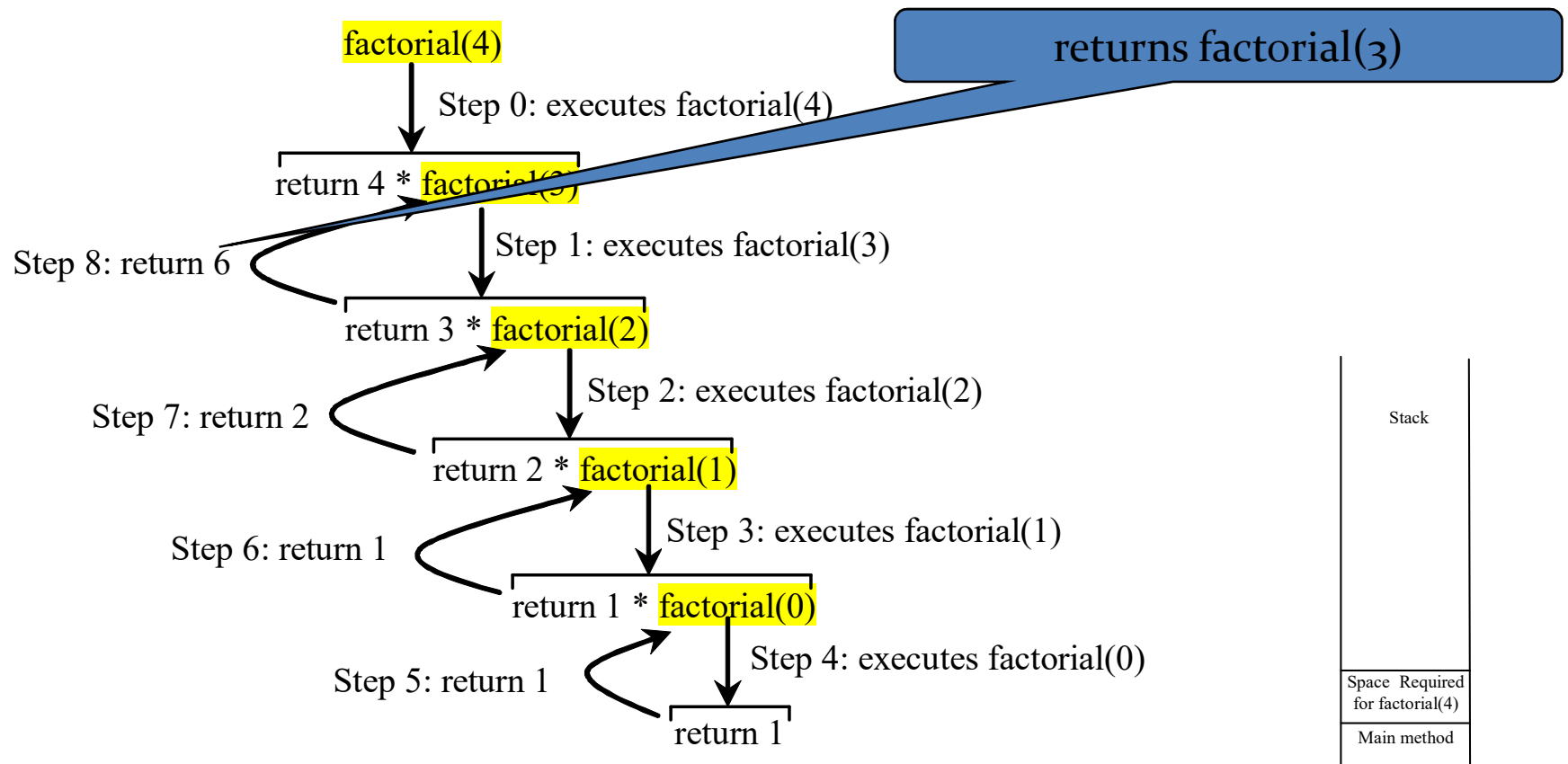
Recursive Factorial



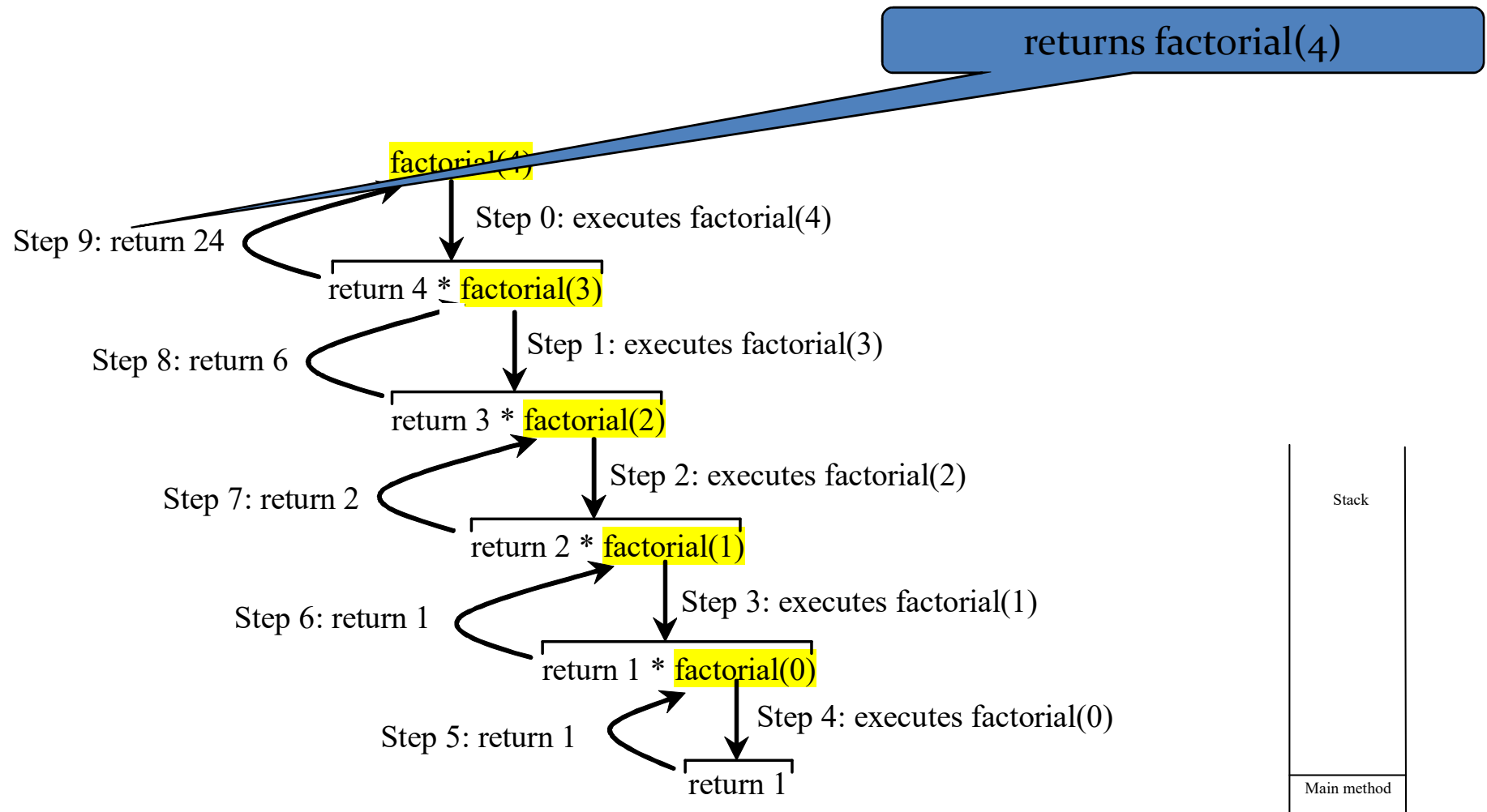
Recursive Factorial



Recursive Factorial



Recursive Factorial



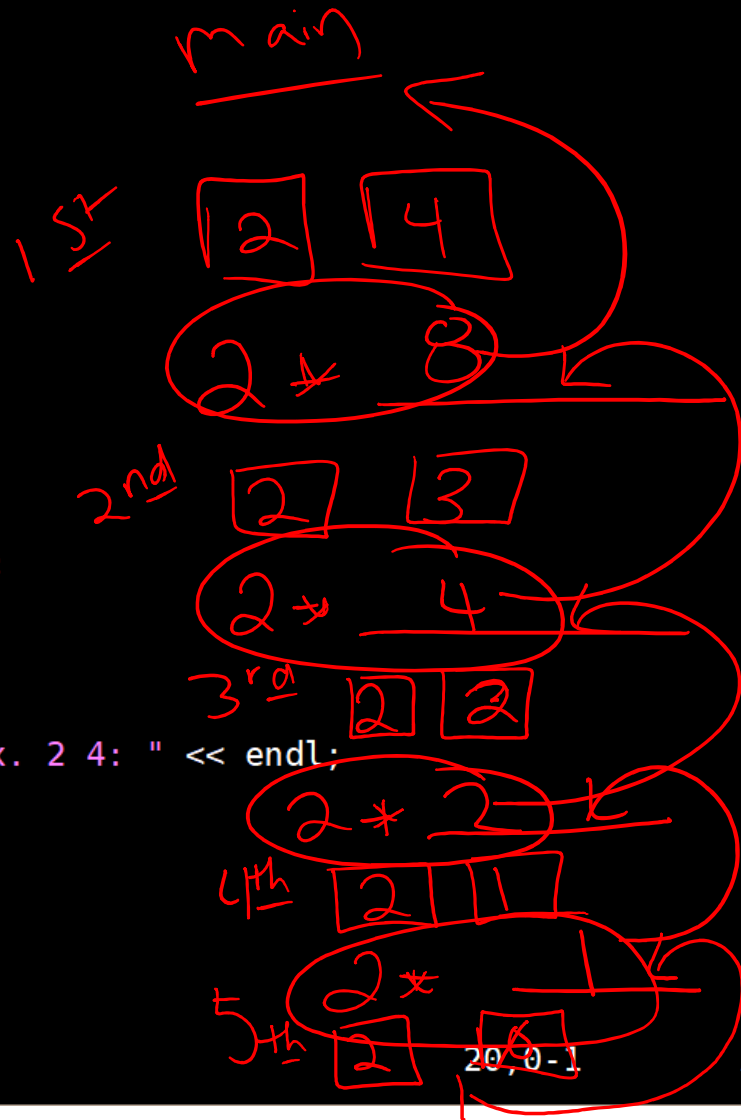
In-class Exercise #4

- Get into groups of 4 – 5.
- Write your own recursive *int pwr()* function that takes two integers as arguments and returns the integer result.
 - What does the function prototype look like?
 - Now, write the function definition...


```

1 #include <iostream>
2
3 using namespace std;
4
5 int power(int base, int exponent) {
6     int p=1;
7     for(int i=0; i<exponent; i++)
8         p*=base;
9
10    return p;
11 }
12
13 int pwr(int base, int exponent) {
14     //base
15     if(exponent==0)
16         return 1;
17     else
18         return base*pwr(base,exponent-1);
19 }
20
21 int main() {
22     int base, exp;
23     cout << "Enter base and exponent, ex. 2 4: " << endl;
24     cin >> base;
25     cin >> exp;
26     cout << power(base, exp) << endl;
27     cout << pwr(base, exp) << endl;
28
29     return 0;
30 }

```

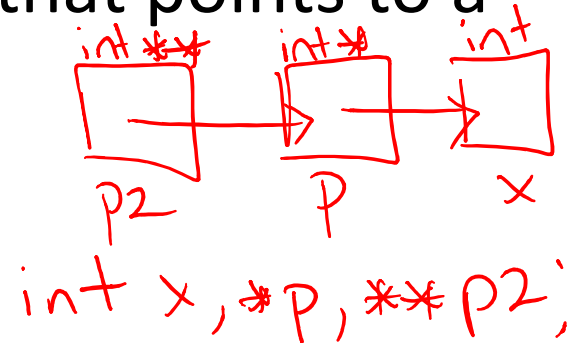


In-class Exercise

Pointers vs. References

- What if you made a pointer (p2) that points to a pointer (p) to an int (x)?

- What would the picture look like?
- Write the code for this picture.



- Can you make this same picture for references? no!

- What if you had two references, r and r2?



*p gives contents of x
*p2 gives contents of p, addr. of x
**p2 gives contents of x