Basic Shape Analysis and Visualization
Topics Today

- Bounding box computation
- Silhouette extraction
- Edge construction
Topics Today

- Bounding box computation
- Silhouette extraction
- Edge construction
Shape Descriptors

• What are features?
Shape Descriptors

• How to compute them?
Shape Descriptors
- Bounding Boxes

- Centers and variances
Center of Mass

\[(x_i, y_i)\]

\[(x_j, y_j)\]
Center of Mass

- Center of Mass
  \[(x_i, y_i)\]

\[
\left( \frac{\sum_{i=1}^{N} x_i}{N}, \frac{\sum_{i=1}^{N} y_i}{N} \right)
\]
Geometric Center

• Geometric center

\((x_i, y_i)\)

\((x_j, y_j)\)

\[\left( \frac{\min x_i + \max x_i}{2}, \frac{\min y_i + \max y_i}{2} \right)\]
Centers

- **Geometric center**
  \[
  \left( \frac{\min x_i + \max x_i}{2}, \frac{\min y_i + \max y_i}{2} \right)
  \]

- **Center of Mass**
  \[
  \left( \frac{\sum_{i=1}^{N} x_i}{N}, \frac{\sum_{i=1}^{N} y_i}{N} \right)
  \]

- Which one is better and why?
Variance

• What are the dominant directions?

\((x_i, y_i)\)
Variance

• What are the dominant directions? 
  \((x_i, y_i)\)

• How are they defined?
  \((x_j, y_j)\)
Variance

- What are the dominant directions?

\[(x_i, y_i)\]

\[
V = \begin{pmatrix}
\sum_{i} (x_i - c_x)(x_i - c_x) & \sum_{i} (x_i - c_x)(y_i - c_y) \\
\sum_{i} (x_i - c_x)(y_i - c_y) & \sum_{i} (y_i - c_y)(y_i - c_y)
\end{pmatrix}
\]
Variance

• What are the dominant directions?
  \((x_i, y_i)\)

• Major, minor eigenvectors give the directions

\((x_j, y_j)\)
Variance

\[ h(p, d) = \begin{bmatrix} d_x & d_y \end{bmatrix} \begin{bmatrix} (x_k - c_x)^2 & (x_k - c_x)(y_k - c_y) \\ (x_k - c_x)(y_k - c_y) & (y_k - c_y)^2 \end{bmatrix} \begin{bmatrix} d_x \\ d_y \end{bmatrix} \]
Variance

\[
\begin{bmatrix}
(x_k - c_x)^2 & (x_k - c_x)(y_k - c_y) \\
(x_k - c_x)(y_k - c_y) & (y_k - c_y)^2
\end{bmatrix}
= \begin{bmatrix}
x_k - c_x \\
y_k - c_y
\end{bmatrix}
\begin{bmatrix}
x_k - c_x & y_k - c_y
\end{bmatrix}
\]

\[
h(p, d) = \begin{bmatrix} d_x & d_y \end{bmatrix}
\begin{bmatrix}
(x_k - c_x)^2 & (x_k - c_x)(y_k - c_y) \\
(x_k - c_x)(y_k - c_y) & (y_k - c_y)^2
\end{bmatrix}
\begin{bmatrix} d_x \\
d_y
\end{bmatrix}
\]
Variance

\[
\begin{bmatrix}
(x_k - c_x)^2 & (x_k - c_x)(y_k - c_y) \\
(x_k - c_x)(y_k - c_y) & (y_k - c_y)^2
\end{bmatrix}
= \begin{bmatrix}
x_k - c_x \\
y_k - c_y
\end{bmatrix}
\begin{bmatrix}
x_k - c_x & y_k - c_y
\end{bmatrix}
\]

\[
h(p, d) = \begin{bmatrix} d_x & d_y \end{bmatrix}
\begin{bmatrix} x_k - c_x \\
y_k - c_y
\end{bmatrix}
\begin{bmatrix} d_x \\
d_y
\end{bmatrix}
\]
Variance

\[ (x_i, y_i) \]

\[ (x_j, y_j) \]

\[ p = (x_k, y_k) \]

\[ h(p, d) = \left( \begin{bmatrix} d_x & d_y \end{bmatrix} \begin{bmatrix} x_k - c_x \\ y_k - c_y \end{bmatrix} \right)^2 \]
Variance

- It is a voting scheme

\[ h(p, d) = \left( \begin{bmatrix} d_x & d_y \end{bmatrix} \begin{bmatrix} x_k - c_x \\ y_k - c_y \end{bmatrix} \right)^2 \]
Bounding Boxes

$$(x_i, y_i)$$

$$(x_j, y_j)$$
Bounding Boxes
Bounding Boxes

\[(x_i, y_i)\]

\[(x_j, y_j)\]
Bounding Boxes

$(x_i, y_i)$

$(x_j, y_j)$
Bounding Boxes

• How do you extend this to 3D?
Another Definition of Variance

\[ h(p, d) = \left( d_x \begin{bmatrix} N_x(p) \\ N_y(p) \end{bmatrix} d_y \right)^2 \]
Any questions?