1. For each of the following systems determine whether they are time-invariant? BIBO Stable? Linear? Causal? Memoryless? Provide reasons for each answer.

(a) $y[n] = n^2 x[n]$
   i. $H\{x[n-N]\} = n^2 x[n-N] \neq (n-N)^2 x[n-N] = y[n-N]$  
      **Not Time Invariant**
   ii. Assume $|x[n]| \leq B = 1$, Investigate $|x[n]| = B = 1$  
       $y[n] = n^2$  
       when $n \to \infty$, $y[n] \to \infty$, not bounded  
      **Not BIBO Stable**
   iii. $y_1[n] = n^2 x_1[n]$, $y_2[n] = n^2 x_2[n]$  
       $y[n] = n^2(ax_1[n] + bx_2[n]) = an^2 x_1[n] + bn^2 x_2[n] = ay_1[n] + by_2[n]$  
      **Linear**
   iv. $y[n]$ depends only on the present input value.  
      **Causal**
   v. $y[n]$ has no dependence on past or future input values.  
      **Memoryless**

(b) $y[n] = e^{x[n]}$
   i. $H\{x[n-N]\} = e^{x[n-N]} = y[n-N]$  
      **Time Invariant**
   ii. Assume $|x[n]| \leq B$,  
       $y[n] = e^B$  
       $y[n]$ is bounded  
      **BIBO Stable**
   iii. $y_1[n] = e^{x_1[n]}$, $y_2[n] = e^{x_2[n]}$  
       $y[n] = e^{ax_1[n]} + e^{bx_2[n]} \neq ae^{x_1[n]} + be^{x_2[n]} = ay_1[n] + by_2[n]$  
      **Not Linear**
   iv. $y[n]$ depends only on the present input value.  
      **Causal**
   v. $y[n]$ has no dependence on past or future input values.  
      **Memoryless**

(c) $y[n] = x[-n]$
   i. $H\{x[n-N]\} = x[-n-N] \neq x[-n+N] = y[n-N]$  
      **Not Time Invariant**
   ii. Assume $|x[n]| \leq B$,  
       $y[n] \leq B$  
       $y[n]$ is bounded  
      **BIBO Stable**
iii. \( y_1[n] = x_1[-n], y_2[n] = x_2[-n] \)
\( y[n] = ax_1[-n] + bx_2[-n] = ay_1[n] + by_2[n] \)

**Linear**

iv. When \( n < 0, \) \( y[n] \) depends on future values.

**Non-Causal**

v. \( y[n] \) depends on either past or future values.

**Not Memoryless**

(d) \( y[n] = \sum_{k=n-n_0}^{n+n_0} x[k] \)

i. \( H\{x[n-N]\} = \sum_{k=n-n_0}^{n+n_0} x[l-N] = \sum_{k=n-n_0-N}^{n+n_0-N} x[l] = y[n-N] \)

**Time Invariant**

ii. Assume \( |x[n]| \leq B, \)

\[ y[n] \leq \sum_{k=n-n_0}^{n+n_0} B = B(2n_0 + 1) \]

\( y[n] \) is bounded

**BIBO Stable**

iii. \( y_1[n] = \sum_{k=n-n_0}^{n+n_0} x_1[k], y_2[n] = \sum_{k=n-n_0}^{n+n_0} x_2[k] \)
\( y[n] = \sum_{k=n-n_0}^{n+n_0} ax_1[k] + bx_2[k] = a \sum_{k=n-n_0}^{n+n_0} x_1[k] + b \sum_{k=n-n_0}^{n+n_0} x_2[k] = ay_1[n] + by_2[n] \)

**Linear**

iv. \( y[n] \) depends on future values.

**Non-Causal**

v. \( y[n] \) depends on past and future values.

**Not Memoryless**

2. For each of the following systems determine whether they are time-invariant? BIBO Stable? Linear? Causal? Provide reasons for each answer.

(a) \( y[n] = \sum_{i=-2}^{n-1} \left( \frac{1}{2} \right)^i x[i+1] \)

i. \( H\{x[n-N]\} = \sum_{i=-2}^{n-1} \left( \frac{1}{2} \right)^i x[i+1-N] \neq \sum_{i=-2}^{n-N-1} \left( \frac{1}{2} \right)^i x[i+1] = y[n-N] \)

**Not Time Invariant**

ii. Assume \( |x[n]| \leq B \)

\[ y[n] \leq \sum_{i=-2}^{n-1} \left( \frac{1}{2} \right)^i B \leq B \left( 4 + 2 + \sum_{i=0}^{n-1} \left( \frac{1}{2} \right)^i \right) \leq B \left( 6 + 1 - \frac{1}{2} \right) \]

when \( n \to \infty, y[n] = B(6 + 2) = 8B, \) bounded

**BIBO Stable**

iii. \( y_1[n] = \sum_{i=-2}^{n-1} \left( \frac{1}{2} \right)^i x_1[i+1], y_2[n] = \sum_{i=-2}^{n-1} \left( \frac{1}{2} \right)^i x_2[i+1] \)
\( y[n] = \sum_{i=-2}^{n-1} a \left( \frac{1}{2} \right)^i x_1[i+1] + b \left( \frac{1}{2} \right)^i x_2[i+1] + a \sum_{i=-2}^{n-1} \left( \frac{1}{2} \right)^i x_1[i+1] + b \sum_{i=-2}^{n-1} \left( \frac{1}{2} \right)^i x_2[i+1] = \)
3. A time-discrete system $H$ is described by:

$$y[n] = \sum_{k=0}^{\infty} (0.5)^k x[n - k]$$

a) Show that $H$ is an LTI system
b) Determine the impulse response $h[n]$
c) Determine whether it is BIBO stable

(a) $H\{x[n - N]\} = \sum_{k=0}^{\infty} (0.5)^k x[n - N - k] = y[n - N] \implies$ Time Invariant

$$y_1[n] = \sum_{k=0}^{\infty} (0.5)^k x_1[n - k], y_2[n] = \sum_{k=0}^{\infty} (0.5)^k x_2[n - k]$$

$$y[n] = \sum_{k=0}^{\infty} (0.5)^k a x_1[n - k] + \sum_{k=0}^{\infty} (0.5)^k b x_2[n - k] = a \sum_{k=0}^{\infty} (0.5)^k x_1[n - k] + b \sum_{k=0}^{\infty} (0.5)^k x_2[n - k] = a y_1[n] + b y_2[n] \implies$ Linear

(b) $y[n] = \sum_{k=0}^{\infty} (0.5)^k \delta[n - k] = 0.5^n$
(c) Assume \(|x[n]| \leq B \implies |y[n]| \leq \sum_{k=0}^{\infty} (0.5)^k B \leq B \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k = B \cdot \frac{1 - 0.5^k}{1 - 0.5} = 2B

**BIBO Stable**

4. (Ex. 2.3) Compute the output \(y(t)\) of the continuous-time LTI system with impulse response \(h(t) = u(t + 1) - u(t - u)\) subjected to the input signal \(x(t) = u(t + 1) - u(t - u)\).

\[
\begin{align*}
\text{if } 1 + t < -1 & \implies t < -2, \ y(t) = 0 \\
\text{if } -1 + t > 1 & \implies t > 2, \ y(t) = 0 \\
\text{if } \begin{cases} 1 + t > -1 \\ -1 + t < -1 \end{cases} & \implies -2 < t < 0 \\
y(t) &= \int_{1}^{1+t} d\tau = t|_{-1}^{1+t} = t + 2 \\
\text{if } \begin{cases} 1 + t > 1 \\ -1 + t < 1 \end{cases} & \implies 0 < t < 2 \\
y(t) &= \int_{-1+t}^{1} d\tau = t|_{-1}^{1+t} = 2 - t \\
y(t) &= \begin{cases} 0 & \text{if } t < 2 \text{ or } t > 2 \\ t + 2 & \text{if } -2 < t < 0 \\ 2 - t & \text{if } 0 < t < 2 \end{cases}
\end{align*}
\]

5. (Ex. 2.4) Determine whether the discrete-time LTI system with impulse response \(h[n] = (-0.9)^n u[n-4]\) is BIBO stable. Is it causal?

\[
\begin{align*}
(a) \quad \sum_{n=-\infty}^{\infty} |h[n]| &= \sum_{n=-\infty}^{\infty} (-0.9)^n u[n-4] \\
&= \sum_{n=4}^{\infty} (-0.9)^n - \left[ (0.9) + (0.9)^2 + (0.9)^3 \right] \\
&= 1 - (0.9)^4 - \left[ (0.9) + (0.9)^2 + (0.9)^3 \right] = 10 - \left[ (0.9) + (0.9)^2 + (0.9)^3 \right] \\
y[n] \text{ is bounded} & \implies \text{BIBO Stable}
\end{align*}
\]
(b) No future inputs needed.

**Causal**

6. (Ex. 2.5) Compute the step response of the LTI system with impulse response $h(t) = e^{-t} \cos(2t)u(t)$.

$$y(t) = s(t) = \int_{-\infty}^{t} e^{-\tau} \cos(2\tau)u(\tau)d\tau$$

$$= \int_{0}^{t} e^{-\tau} \cos(2\tau)d\tau$$

$$= e^{-\tau}/5 \left(-\cos(2\tau) + 2\sin(2\tau)\right)\big|_{0}^{t}$$

$$= e^{-t}/5 \left(-\cos(2t) + 2\sin(2t)\right) + 1/5$$

7. (Ex. 2.6) Compute the output $y(t)$ of the continuous-time LTI system with impulse response $h(t)$ for an input signal $x(t)$ as depicted below:

$$y(t) = \begin{cases} 0 & \text{if } t < -1 \\ e^{-(t+1)} - 1 & \text{if } -1 < t < 3 \\ e^{-(t-3)} - e^{-(t+1)} & \text{if } t > 3 \end{cases}$$