\[ h[n] = s[n] + 0.5 s[n-100] + 0.25 s[n-200] \]

\[ y[n] = x[n] * h[n] = x[n] * (s[n] + 0.5 s[n-100] + 0.25 s[n-200]) \]

\[ = x[n] + 0.5 x[n-100] + 0.25 x[n-200] \]

Echo example

---

**Interconnection of LTI systems**

**Given:**

\[
\begin{align*}
    h_1(t) & \xrightarrow{\mathcal{H}_1} \mathcal{H}_1 \\
    h_N(t) & \xrightarrow{\mathcal{H}_N} \mathcal{H}_N
\end{align*}
\]

\[ \implies \text{form a bigger system} \xrightarrow{\mathcal{H}} \]

**Question:** How is \( h(t) \) related to \( h_1(t) \cdots h_N(t) \)?

* Parallel Connection

![Image of parallel connection diagram]
Interconnection of LTI systems (cont.)

\[ y(t) = x(t) \ast h_1(t) + x(t) \ast h_2(t) = x(t) \ast (h_1(t) + h_2(t)) \]

Distribution property of convolution process:

Interconnection of LTI systems (cont.)

- **Cascade Connection**

\[ x(t) \rightarrow h_1(t) \rightarrow z(t) \rightarrow h_2(t) \rightarrow y(t) \Rightarrow x(t) \rightarrow h_1(t) \ast h_2(t) \rightarrow y(t) \]
Interconnection of LTI systems (cont.)

- Associative Property (Same for DT)

- Commutative Property (Same for DT)

E: Example 2.11, \textit{IEE}89: See figure below. Find the impulse response \( h[n] \) of the overall system.

\[
\begin{align*}
  h_1[n] &= u[n] \\
  h_2[n] &= u[n + 2] - u[n] \\
  h_3[n] &= \delta[n - 2] \\
  h_4[n] &= \alpha^n u[n]
\end{align*}
\]

\[
h[n] = (h_1[n] + h_4[n]) \ast h_2[n] - h_4[n]
\]

\[
= \left( (v[n] + v[n + 2] - v[n]) \ast \delta[n - 2] \right) - \alpha^n v[n]
\]

\[
= v[n] - \alpha^n v[n] \\
\]

\[
y[n] = \sum_{k=0}^{n} x[k] h[n-k]
\]
Interconnection of LTI systems (cont.)

E: An interconnection of LTI system is depicted in the figure below. \( h_1[n] = (\cdot)^n u[n+2] \), \( h_2[z] = \delta[n] \), and \( h_3[n] = u[n-1] \). Find the impulse response \( h[n] \) of the overall system.

\[
\begin{align*}
    h[n] &= h_1[n] \star (h_2[n] + h_3[n]) \\
    &= (-1)^n v[n+2] \star (\delta[n] + v[n-1]) \\
    &= (-1)^n v[n+2] - (-1)^n u[n+2] \star v[n-1]
\end{align*}
\]

LTI SYSTEM PROPERTIES & IMPULSE RESPONSE

System properties (Chapter 1)

- Stability (BIBO)
- Memory (depend on current input only)
- Causality (does not depend on future inputs)
- Linearity
- Time invariance \( \{ \) LTIC

\{ Memoryless, Stable, LTI \}

Memoryless, LTI

For LTI systems:

\[
    h(t) \quad \text{Completely determine} \quad \text{Input-output behaviors}
\]

\[
    h[n] \quad \text{Input-output behaviors}
\]

Thus, stability, memory, causality are related to \( h(t)/h[n] \).
a) If an LTI system is MEMORYLESS

\[ h[k] = c \delta[k] \quad \text{iff} \quad h(\tau) = c \delta(\tau) \]

Proof:
\[ y[n] = x[n] \ast h[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k] \]
\[ = \sum_{k=-\infty}^{\infty} h[k] x[n-k] = \ldots + h[-1] x[n+1] + h[0] x[n] + h[1] x[n-1] + \ldots \]

By definition of memoryless property, \( y[n] \) can only depend on \( x[n] \) not \( x[n+k] \) for \( k \neq 0 \),

Therefore \( h[n] = 0 \) for all \( n \neq 0 \)

\[ \Rightarrow h[n] = c \delta[n] \]

b) If an LTI system is CAUSAL:

\[ \begin{align*}
\text{DT:} & \quad h[k] = 0 \text{ for } k < 0 \\
\text{CT:} & \quad h(\tau) = 0 \text{ for } \tau < 0 
\end{align*} \]

Proof:
\[ y[n] = \sum_{k=-\infty}^{\infty} h[k] x[n-k] = \ldots + h[-1] x[n+1] + h[0] x[n] + h[1] x[n-1] + \ldots \]

By definition of causality,
\[ \Rightarrow h[n] = 0 \text{ for } n < 0. \]
c) If an LTI system is BIBO STABLE:

Proof:

\[
|y[n]| = |x[n] \ast h[n]| = \left| \sum_{k=-\infty}^{\infty} h[k] x[n-k] \right| \leq \sum_{k=-\infty}^{\infty} |h[k]| |x[n-k]| \leq B \sum_{k=-\infty}^{\infty} |h[k]| \Rightarrow \text{converges}
\]

\[
\sum_{k=-\infty}^{\infty} |h[k]| < \infty \quad \text{and} \quad \int_{-\infty}^{\infty} |h(\tau)| d\tau < \infty \quad |x[n]| \leq B
\]