1. 

a) 

\[ |X(k)| = \left| \frac{k}{1+jk} \right| = \left| \frac{k(1-jk)}{(1+jk)(1-jk)} \right| = \left| \frac{k - jk^2}{1 + k^2} \right| = \sqrt{\frac{k^2}{(1+k^2)^2} + \frac{k^4}{(1+k^2)^2}} = \sqrt{\frac{k^2}{(1+k^2)}} = \frac{k}{\sqrt{1+k^2}} \]

At frequency of 440 Hz, \( k = \frac{440}{440} = 1 \)

Thus \( |X(1)| = \frac{1}{\sqrt{2}} \)

b) 880 Hz is NOT a multiple of 440 Hz, thus, the energy level is 0.
2. $h_1$ and $h_2$ are cascaded, thus $H_{12}(j\omega) = H_1 \cdot H_2$

$H_1$ and $h_3$ are parallel, thus

$Y(j\omega) = X(j\omega) (H_1 \cdot H_3) + X(j\omega) H_3$

$H_1(j\omega)$:

$H_2(j\omega)$:

Thus, $H_{12}(j\omega)$.

$H_3(j\omega)$:
Now, by Euler's formula
\[
X(t) = \frac{e^{j\frac{3\pi}{4}t} - e^{-j\frac{3\pi}{4}t}}{j} + \frac{3}{2} \left( e^{j\frac{3\pi}{4}t} + e^{-j\frac{3\pi}{4}t} \right)
\]

For \( H(m)(jw) \), only the frequency component of \( e^{j\frac{3\pi}{2}t} \) can pass. Thus
\[
Y_{111}(t) = |w| \frac{e^{j\frac{3\pi}{4}t}}{j} - |w| \frac{e^{-j\frac{3\pi}{4}t}}{j}
\]
\[
= \frac{3\pi}{4} \left( \frac{e^{j\frac{3\pi}{4}t} - e^{-j\frac{3\pi}{4}t}}{j} \right)
\]
\[
= \frac{3\pi}{4} \cdot 2 \sin \frac{3\pi}{4} t = \frac{3\pi}{2} \sin \left( \frac{3\pi}{4} t \right)
\]

For \( H(m)(jw) \), only the frequency component of \( e^{j\frac{3\pi}{2}t} \) can pass. Thus
\[
Y_{011}(t) = -1 \left( \frac{3}{2} \left( e^{j\frac{3\pi}{4}t} + e^{-j\frac{3\pi}{4}t} \right) \right)
\]
\[
= -3 \cos \left( \frac{3\pi}{2} t \right)
\]

\[
Y = Y_{011}(t) + Y_{111}(t) = \frac{3\pi}{2} \sin \left( \frac{3\pi}{4} t \right) - 3 \cos \left( \frac{3\pi}{2} t \right)
\]
3. (a) For \( 2 \cos \frac{3\pi}{5} k \), we need to find minimum integer \( N \) such that
\[
\frac{3\pi}{5} N_1 = 2\pi l_1, \quad N_1 = \frac{10}{3} l_1.
\]
Thus, \( l_1 = 3 \), \( N_1 = 10 \).

Similarly, for \( \sin \frac{3\pi}{5} k \):
\[
N_2 = \frac{4}{3} l_2, \quad l_2 = 3, \quad N_2 = 4
\]
Thus, \( N = \text{LCM}(10, 4) = 20 \).

(b) by Euler's formula,
\[
X(k) = \frac{1}{2j} \left( e^{j\frac{3\pi}{5}k} - e^{-j\frac{3\pi}{5}k} \right) + e^{j\frac{\pi}{5}k} + e^{-j\frac{3\pi}{5}k}
\]
Since \( N = 20 \), \( \Omega = \frac{2\pi}{N} = \frac{\pi}{10} \),
\[
X(k) = \frac{1}{2j} e^{j\frac{3(15)\pi}{5}k} - \frac{1}{2j} e^{-j\frac{3(15)\pi}{5}k}
+ e^{j\frac{6\pi}{5}k} + e^{j(-6)\frac{\pi}{5}k}
= \frac{1}{N} \sum_{n=0}^{N-1} X(n) e^{-j2\pi \frac{n}{N}}
\]
by comparison:
\[
X(n) = \begin{cases} 
2^6, & n \equiv 1 \pmod{16} \\
\frac{10}{3} & n \not\equiv 15 \pmod{20} \\
\frac{10}{3} & n = 15 \\
0 & \text{otherwise}
\end{cases}
\]
Since \(-15\) is outside period, add \( N \)\( = -15 + N = -15 + 20 = 5 \Rightarrow n = 5 \).
4. \( Z(t) \).

The graph assuming \( a > 1 \), when \( a < 1 \), the graph is decreasing.

\( X(t) \) is simply shifting the "block" of \( Z(t) \) one by \((k+1)T\) in and add all the "shifted block" together.

Thus \( \Phi = K+1 \)

b) We need to find \( FS \), only consider one period:

\[
X(k) = \frac{1}{T} \int_{0-T}^{T} x(t) e^{-j\omega_k t} dt
\]

\( \omega \) is a small constant \((\omega < 1)\) such that the integral range only include the impulses from 0 to \( K \).

Then:

\[
X(k) = \frac{1}{K+1} \sum_{\xi = 0}^{K} \int_{-\xi}^{K+1-\xi} a^i \delta(t-i)e^{-j\frac{2\pi}{K+1}t} dt
\]

\[
= \frac{1}{K+1} \sum_{\xi = 0}^{K} \int_{-\xi}^{K+1-\xi} a^i \delta(t-i) e^{-j\frac{2\pi}{K+1}t} dt
\]
\[ = \frac{1}{K+1} \left( \sum_{i=0}^{K} a^i e^{-j \frac{2\pi k i}{K+1}} \right) \]

Notice this is a geometric series.

\[ X(k) = \frac{1}{K+1} \left( \sum_{i=0}^{K} (ae^{\frac{2\pi k i}{K+1}})^i \right) \]

\[ = \frac{1}{K+1} \left( \frac{1 - ae^{\frac{2\pi k}{K+1}}}{1 - ae^{\frac{2\pi k}{K+1}}} \right) \]