1. In the terminated two-port shown below, the input impedance is $Z_1$.

a. At what frequencies is $Z_1 = R$ for all values of $R$?

b. For what value of $R$ does $Z_1 = R$ hold at all frequencies? Why?

\[ v_1 = z_{11} i_1 + z_{12} i_2 \]
\[ v_2 = z_{21} i_1 + z_{22} i_2 \]
\[ v_2 = -i_2 R \]
\[ Z_1 = \frac{v_1}{i_1} = z_{11} - \frac{z_{12} z_{21}}{z_{22} + R} \]

\[ Z_{in} = \frac{R}{\frac{1}{Z_{22}} + \frac{1}{R}} \]

1.
\[ z_{11} = \frac{1}{2} \left[ \frac{1}{sC} + sL \right] = Z_{22} \]
\[ z_{12} = \frac{1}{2} \left[ \frac{1}{sC} - sL \right] = Z_{21} \text{ reciprocal} \]
\[ Z_1 = z_{11} - \frac{z_{12} z_{21}}{z_{22} + R} = R \left( \frac{z_{11} + (L/C)/R}{z_{11} + R} \right) \]

a. By inspection, $z_{11} \to 0$ for $s = 0$ or $s \to \infty$, so $Z_1 \to R$ for $f = 0, \infty$

b. If $(L/C)/R = R$, or $L/C = R^2$, $Z_1(s) = R \forall s$, $R = 1.732$ kΩ

Or

b. $R = z_{11} - \frac{z_{12}^2}{z_{11} + R}$, $R - z_{11} = -\frac{z_{12}^2}{R + z_{11}}$

$R^2 - z_{11}^2 = -z_{12}^2$, $R = 0L$, $1/(3C) = 1/L$
2. What is the input impedance of the circuit shown below? What are its possible applications?

\[ Z \rightarrow \]

\[ Y_{in} \rightarrow \]

2. The input admittance of the active branch

\[ I = -\frac{V}{R_i R_L} \]

\[ Y_{in} = -\frac{R_2}{R_1 R_L} \]

Negative conductance may cancel \(1/R\) → oscillator, high-Q filter

Overall

\[ Z_i = \frac{1}{1/sL + sC + 1/R - R_2 (R_1 R_L)} \]

Q booster, oscillator
3. (Extra credit) Find all node voltages in the circuit shown below.

\[ E = 2V \]

\[ 1 \rightarrow 2/16 = 1/8 \text{ V} \]

So the node voltages are

\[ 2, 1, 0.5, 0.25 \text{ and } 0.125 \text{ V} \]

Or: Resistance after node = 2R

\[ 0 \text{ before } \rightarrow = R \]

So, at second node, \( V_2 = V_1/2 = E/2 \)

at 3rd node, \( V_3 = E/4 \), etc.

R - 2R DAC!