1. Problem 1: (20 pts) Let \( x(t) = u(1 - t) \) and \( h(t) = t(u(t) - u(t - 2)) \).
   (a) Sketch \( h(\tau), x(\tau), x(t - \tau) \) and carefully label the values on the axes. (6 pts)
   (b) Determine \( y(t) = x(t) \ast h(t) \) by performing graphical convolution. No need to sketch \( y(t) \). (14 pts)

2. Problem 2: (30 pts) Using either the definition or inspection method,
   (a) Compute the appropriate Fourier representation for \( x[n] = e^{-\frac{j\pi}{4} n} \cos\left(\frac{\pi n}{4}\right) + \sin\left(\frac{\pi n}{3}\right) \). (15 pts)
   (b) Let \( x(t) \) be a periodic signal with the fundamental angular frequency \( \frac{2\pi}{3} \). Find \( x(t) \) given that its Fourier representation is \( X[k] = \left(-\frac{1}{2}\right)^k u[k - 2] \). (15 pts)

3. Problem 3: (20 pts)

   \[
   x(t) \xrightarrow{h_1} y(t) \xrightarrow{h_2} z(t)
   \]

   Figure 1: Problem 3

   An LTI system consisting of two LTI subsystems are shown in the Figure 1. Let the frequency responses of the two subsystems \( h_1 \) and \( h_2 \) be defined as:

   \[
   H_1(j\omega) = \begin{cases} 
   |\omega| \frac{\pi}{3} \leq |\omega| < \frac{2\pi}{3} \\
   0 & \text{otherwise}
   \end{cases}, \quad H_2(j\omega) = \begin{cases} 
   w^2 & \omega > \frac{\pi}{3} \\
   0 & \text{otherwise}
   \end{cases}
   \]

   Let \( x(t) = 4\cos^2\left(\frac{\pi t}{4}\right) + e^{-j\pi t} \sin\left(\frac{\pi t}{3}\right) \) be the input into the system.

   (a) Determine \( n, c_i \)'s and \( \omega_i \)'s such that \( x(t) = \sum_{i=1}^{n} c_i e^{j\omega_i t} \) (10 pts)
   (b) Use the property of LTI system with eigenfunction \( e^{j\omega t} \) as input to determine the output \( y(t) \) from the system \( h_1 \). (DO NOT USE OTHER METHOD) (5pts)
   (c) Determine the output \( z(t) \) from the system \( h_2 \). (5pts)

4. Problem 4: (15 pts) Let

   \[
   m(t) = e^{-2t}u(t - 3), \quad n(t) = \cos(\pi t)m(t), \quad o(t) = \delta(t - 1) * n(t).
   \]

   Use the properties of FT and the well-known FT pairs in the table provided to find the following Fourier representations:

   (a) \( M(j\omega) \) (5 pts)
   (b) \( N(j\omega) \) (5 pts)
   (c) \( O(j\omega) \) (5 pts)
5. **Problem 5: (15pts)** A system is characterized by the following input-output relationship:

\[ y[n] = \sum_{i=n-2}^{\infty} 2^{i-n} x[i+1] \]

(a) Show that the system is an LTI system (5 pts)

(b) Determine the impulse response \( h[n] \) of the system. (5 pts)

(c) Is \( h[n] \) BIBO stable? Justify your answers. (5 pts)

6. **Bonus Problem: (5 pts)** Let

\[ x(t) = t^2(u(t) - u(t - 1)). \]

Determine \( X(0) \).