1

Convolution integral

- For CT case.
- Recall DT case:

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

Note: Weighed SUM of time-shifted impulses. Similarly,

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau$$

Note: Weighted superposition of time-shifted impulses.

$$x(t) \xrightarrow{\mathcal{H}} y(t)$$

Convolution integral (cont.)

$$\begin{aligned} y(t) &= \mathcal{H}\left\{\int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau\right\} \quad \text{linear operators} \\ &= \int_{-\infty}^{\infty} x(\tau)\mathcal{H}\{\delta(t-\tau)\}d\tau \end{aligned}$$

$$\delta(t-\tau) \xrightarrow{\mathcal{H}} h(t-\tau)$$

Thus,

$$\boxed{y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau}$$

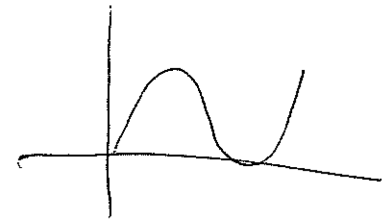
Note:

$$x(t) * h(t) = h(t) * x(t)$$
$$\delta(t) * h(t) = h(t)$$
$$\delta(t-t_0) * h(t) = h(t-t_0)$$

Convolution Integral

E: RADAR range measurement: RADAR-Radio Detection And Ranging:

$$\text{Tx: } x(t) = \begin{cases} \sin(w_c t), & 0 \le t \le T_0 \\ 0, & o.w. \end{cases}$$



Typically,

$$h(t) = \alpha\delta(t-\beta),\ \beta > 0$$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau = \int_{-\infty}^{\infty} x(\tau)\,\alpha\,\delta(t-\beta)\,d\tau$$

$$= \alpha x(t-\beta)$$ (property of $\delta(t)$)

$$\boxed{y(t) = \begin{cases} \alpha \sin(w_c(t-\beta)) & 0 \le t-\beta \le T_0 \\ 0 & o.w. \end{cases}}$$

Convolution integral evaluation procedure

1. Graph $x(t)$ and $h(t)$

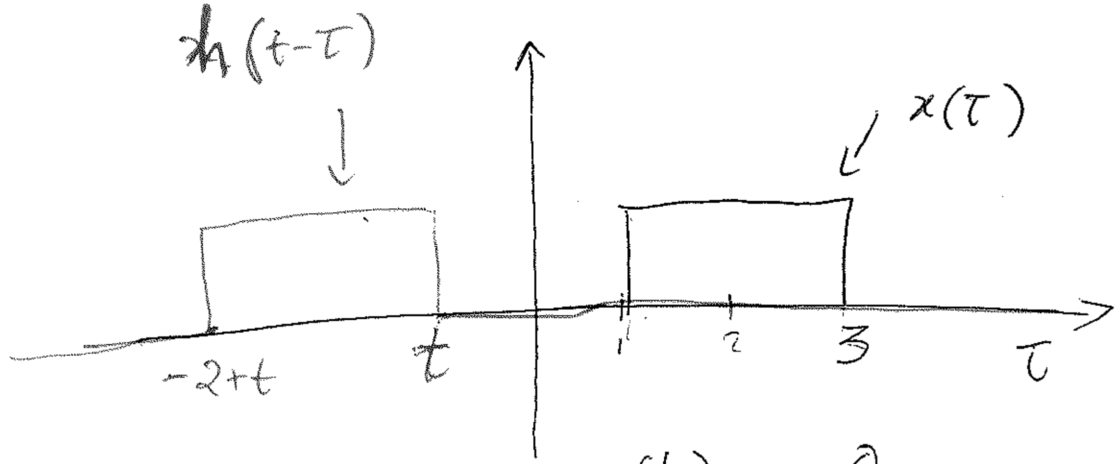
2. Time reverse $h(\tau) \Rightarrow h(-\tau)$

3. Time shift $h(-\tau)$ by $t \Rightarrow h(t-\tau)$

4. For a specific value of t, form product $x(\tau)h(t-\tau)$

5. Integrate $x(\tau)h(t-\tau) \Longrightarrow$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$



case 1: $t < 1 \Rightarrow y(t) = 0$

case 2: $y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

$1 \leq t \leq 3$

$y(t) = \int_{1}^{t} (1)(1)\, d\tau = t - 1$

case 3: $1 \leq -2+t \leq 3 \Rightarrow 3 \leq t \leq 5$

$y(t) = \int_{-2+t}^{3} (1)(1)\, d\tau = 5 - t$

case 4: $3 \leq -2+t \Rightarrow t \geq 5$

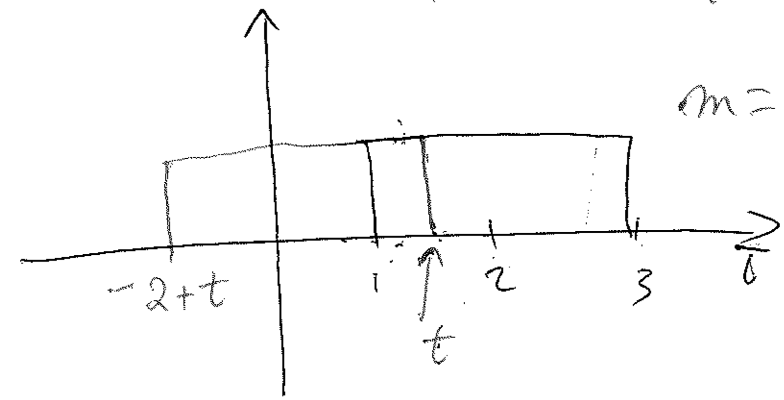
$y(t) = 0$

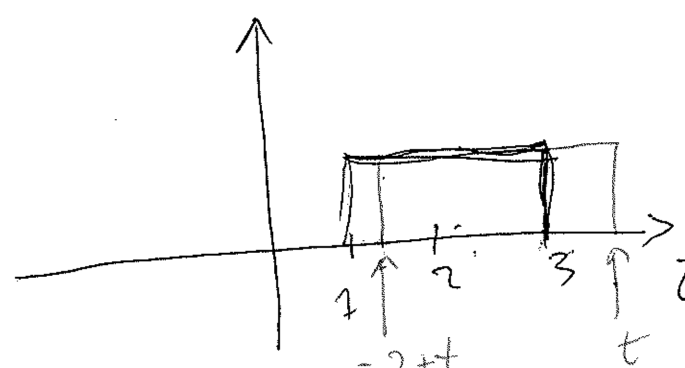
$\delta[n-1]$

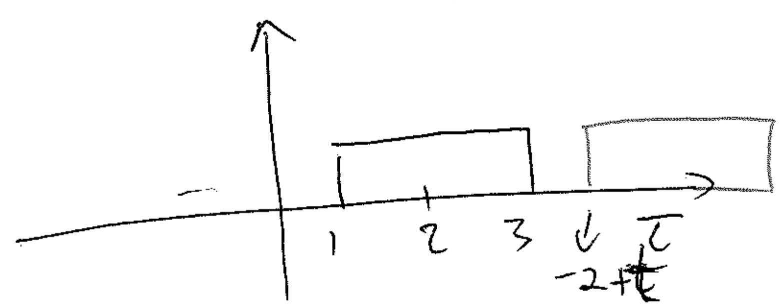
$n = m$

$\cancel{\delta[n]} = \delta[m]$

$m = n-1$





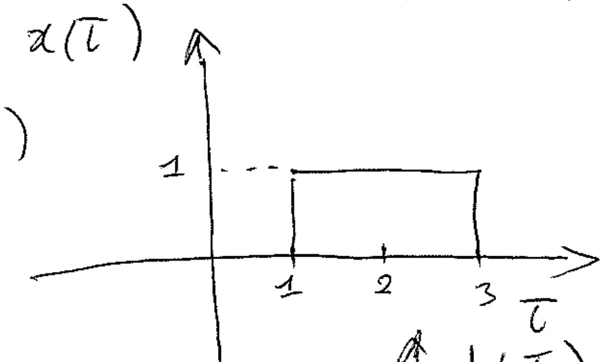


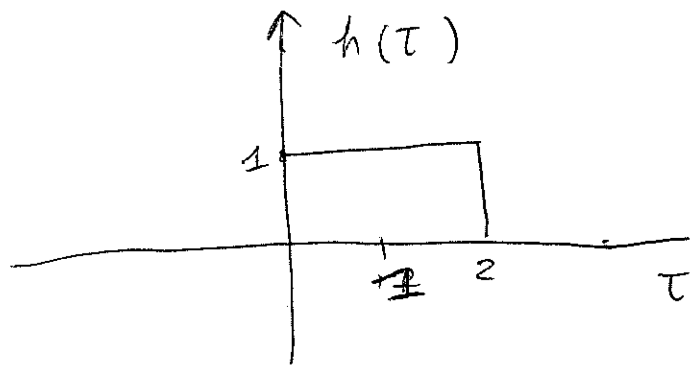
Convolution integral evaluation procedure (cont.)

Example:

$$x(t) = u(t-1) - u(t-3) \xrightarrow{h(t)=u(t)-u(t-2)} y(t) = ??$$

1)





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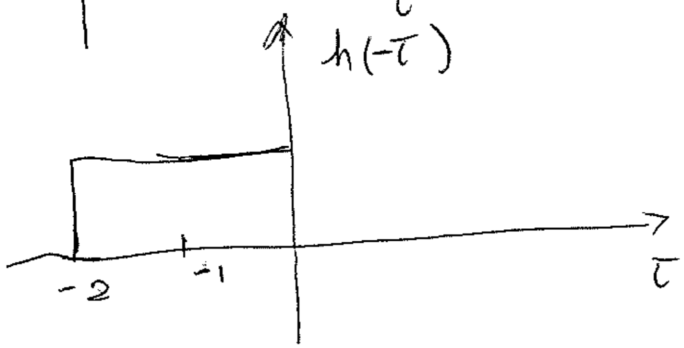

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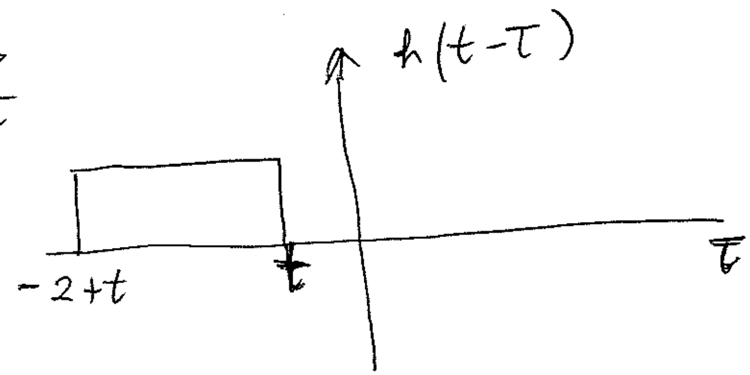

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