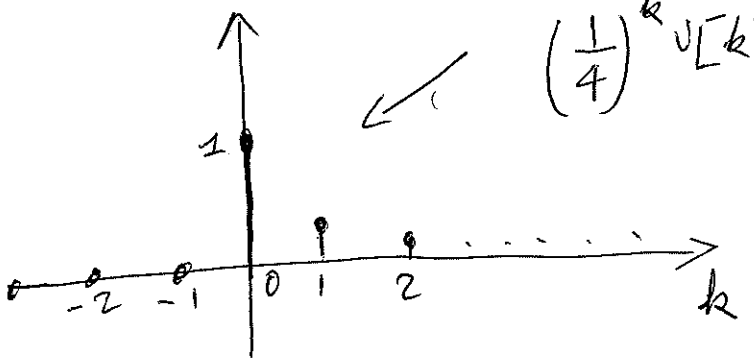
Graphical Convolution in discrete time.

ex:

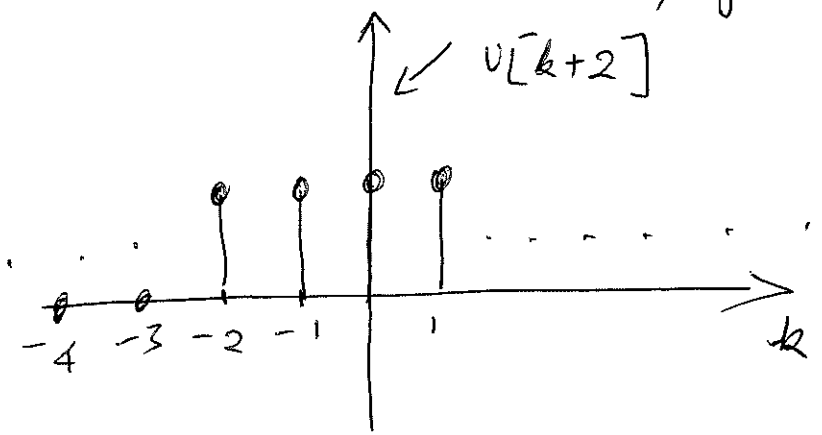
$$y[n] = \left(\frac{1}{4}\right)^n u[n] * u[n+2]$$

(homework 3, problem 1a)

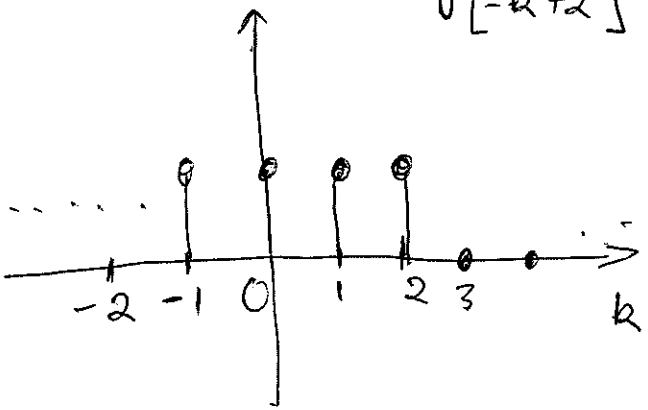
$\left(\frac{1}{4}\right)^k u[k]$

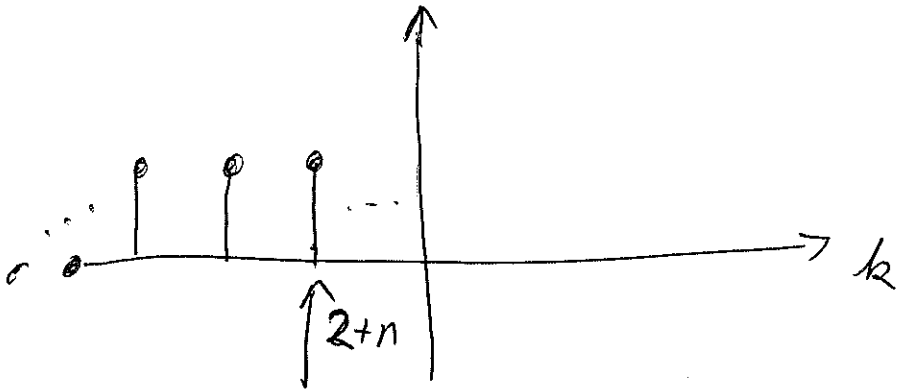


$u[k+2]$



$u[-k+2]$

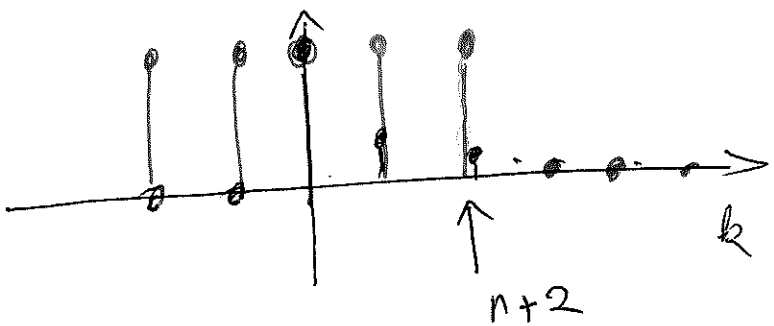




Case 1: $2+n < 0 \Rightarrow n < -2 \Rightarrow y[n] = 0$

Case 2: $y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$

$$= \sum_{k=0}^{n+2} \left(\frac{1}{4}\right)^k = \frac{1-\left(\frac{1}{4}\right)^{n+3}}{1-\frac{1}{4}}$$



$$\sum_{k=0}^{n} a^k = \frac{1-a^{n+1}}{1-a}$$

If $|a| < 1$ then $\sum_{k=0}^{\infty} a^k = \frac{1}{1-a}$

$$y[n] = \begin{cases} 0 & n < -2 \\ \dfrac{1-\left(\frac{1}{4}\right)^{n+3}}{\frac{3}{4}} & n \geq -2 \end{cases}$$

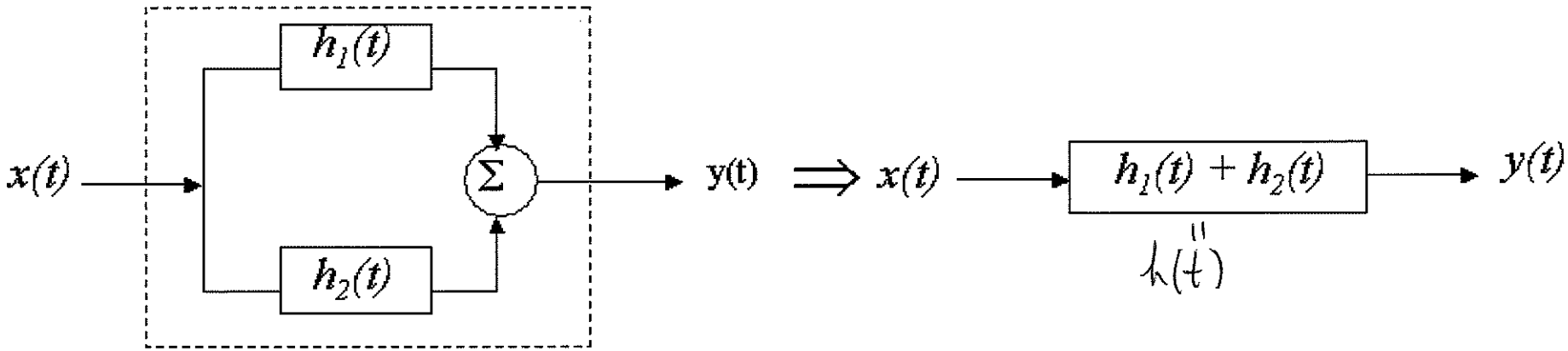
Interconnection of LTI systems

Given:

$$\left.\begin{array}{cc} h_1(t) & \xrightarrow{\mathcal{H}_1} \\ \vdots & \vdots \\ h_N(t) & \xrightarrow{\mathcal{H}_N} \end{array}\right\} \Longrightarrow \text{form a bigger system} \xrightarrow{\mathcal{H}}$$

Question: How is $h(t)$ related to $h_1(t) \cdots h_N(t)$?

- Parallel Connection



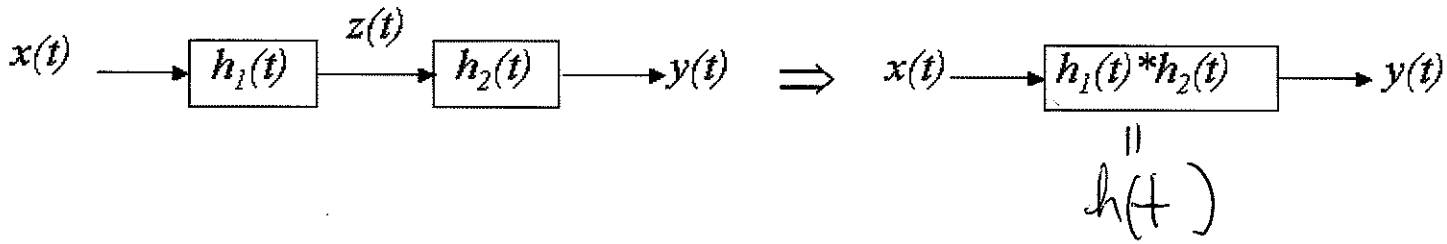
Interconnection of LTI systems (cont.)

you will prove this (or look at the book)

Distribution property of convolution process:

Interconnection of LTI systems (cont.)

- Cascade Connection



you will do this.

↓

prove

Interconnection of LTI systems (cont.)

Let $\eta = \tau - \nu,\ d\eta = d\tau$ (for fixed ν). Then,

Interconnection of LTI systems (cont.)

- Associative Property (Same for DT)

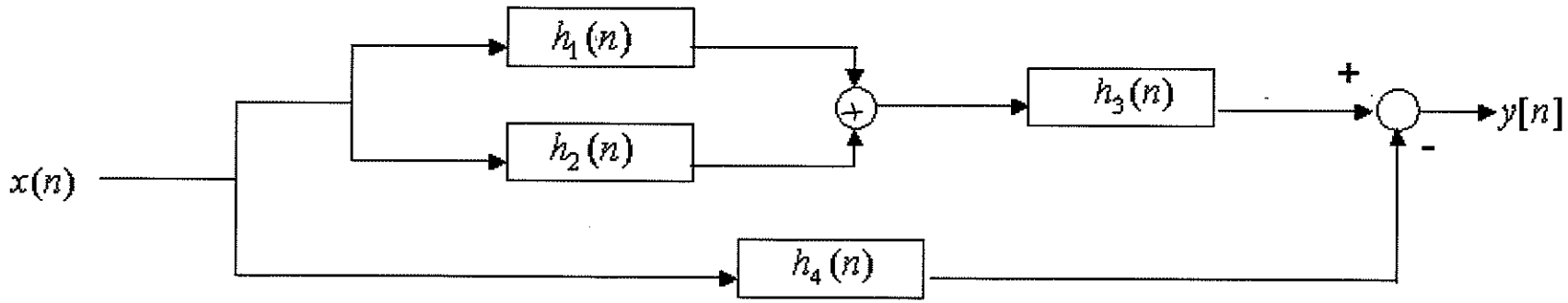
$$x[n] * (y[n] + z[n]) = x[n] * y[n] + x[n] * z[n]$$

(parallel system)

- Commutative Property (Same for DT)

$$x[n] * y[n] = y[n] * x[n]$$

E: Example 2.11, p_{130}: See figure below. Find the impulse response $h[n]$ of the overall system.



Interconnection of LTI systems (cont.)

$$\begin{cases} h_1[n] = u[n] \\ h_2[n] = u[n+2] - u[n] \\ h_3[n] = \delta[n-2] \\ h_4[n] = \alpha^n u[n] \end{cases}$$

$$h[n] = \left(h_1[n] + h_2[n]\right) * h_3[n] - h_4[n]$$

$$h[n] = \left(u[n] + u[n+2] - u[n]\right) * \delta[n-2] - \alpha^n u[n]$$

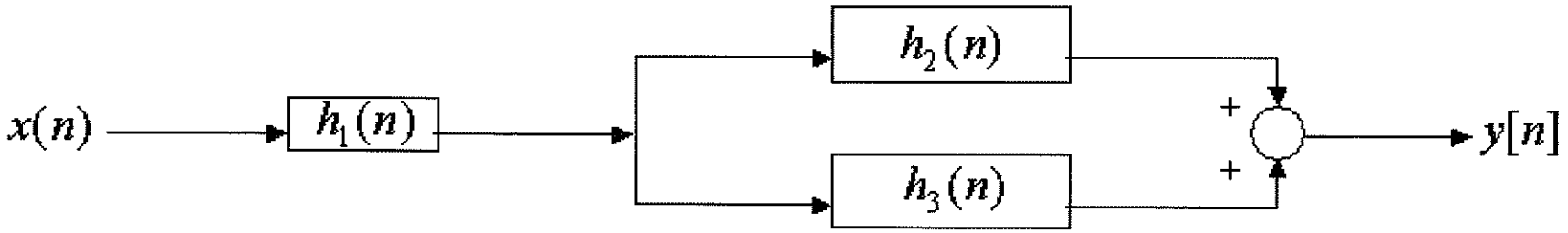
$$= u[n+2] * \delta[n-2] - \alpha^n u[n]$$

$$= u[n-2+2] - \alpha^n u[n]$$

$$u[n] - \alpha^n u[n]$$

Interconnection of LTI systems (cont.)

E: An interconnection of LTI system is depicted in the figure below. $h_1[n] = (1)^n u[n+2]$, $h_2[n] = \delta[n]$, and $h_3[n] = u[n-1]$. Find the impulse response $h[n]$ of the overall system.



you will do this.

a). If an LTI system is MEMORYLESS $\Longleftrightarrow$ *iff* $h[k] = c\delta[k]$, $h(\tau) = c\delta(\tau)$

Proof: $y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$

$\Rightarrow$ $h[n-k] = c\delta[n]$, otherwise

$y[n]$ will depend on $x[n+a]$ where $a \neq 0$

b) If an LTI system is CAUSAL: $\Longleftrightarrow$ $\begin{array}{l}\text{DT}: h[k]=0 \text{ for } k<0 \\ \text{CT}: h(\tau)=0 \text{ for } \tau<0\end{array}$

Proof:

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k] = \ldots + h[-2]\, x[n+2]$$
$$+ h[-1]\, x[n+1] + h[0]\, x[n]$$
$$+ h[1]\, x[n-1] + h[2]\, x[n-2] + \ldots$$

$\Rightarrow$ $h[k]=0$ $k<0$ for $y[n]$ to depend only on $x[n]$, $x[n-1]$, $x[n-2]$, ...