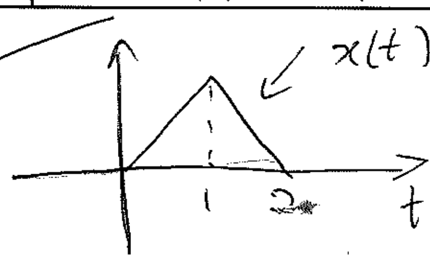
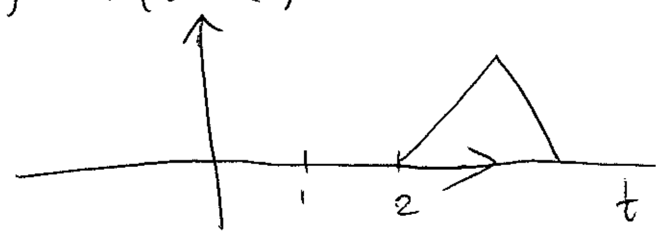
Basic operations on signals

Operation	CT	DT	Note
Amplitude scaling	$y(t) = cx(t)$	$y[n] = cx[n]$	$c > 1$: gain $c < 1$: atten
Addition	$y(t) = x_1(t) + x_2(t)$	$y[n] = x_1[n] + x_2[n]$	
Multiplication	$y(t) = x_1(t)x_2(t)$	$y[n] = x_1[n]x_2[n]$	
Differentiation	$y(t) = \frac{d}{dt}x(t)$	(NO DT case)	
Integration	$y(t) = \int_{-\infty}^{t} x(\tau)d\tau$	(NO DT case)	
Time scaling	$y(t) = x(at)$ $\begin{cases} a > 1: & \text{compression} \\ a < 1: & \text{expansion} \end{cases}$	$y[n] = x[kn]$ $k > 0$ and integer only	
Reflectio (time reversal)	$y(t) = x(-t)$	$y[n] = x[-n]$	
Time shifting	$y(t) = x(t - t_0)$ $\begin{cases} t_0 > 0: & \text{right shift} \\ t_0 < 0: & \text{left shift} \end{cases}$	$y[n] = x[n - n_0]$ $\begin{cases} n_0 > 0: & \text{right shift} \\ n_0 < 0: & \text{left shift} \end{cases}$	
Combination	$y(t) = x(at - t_0)$	$y[n] = x[kn - n_0]$	

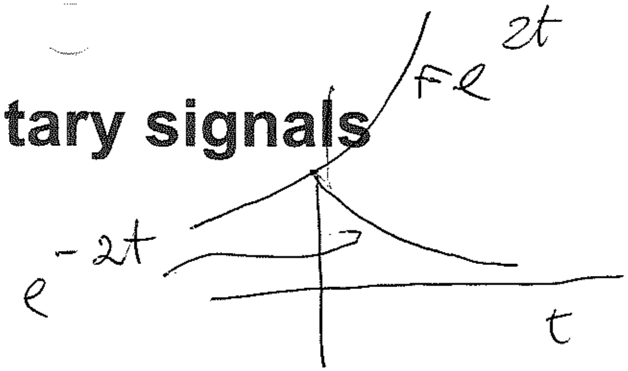


Shift first
scale ~~later~~ later

$y(t) = x(t-2)$



Elementary signals



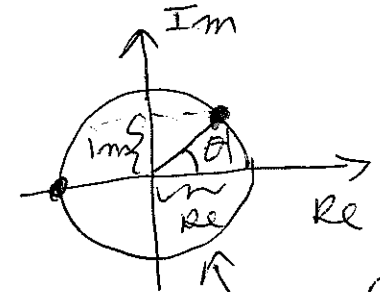
1. Exponential

CT	DT
$x(t) = Be^{at}$, a, B real $\begin{cases} a < 0: & \text{decaying} \\ a > 0: & \text{growing} \\ a = 0: & \text{DC} \end{cases}$	$x[n] = Br^n$ $\begin{cases} 0 < r < 1: & \text{decaying} \\ r > 1: & \text{growing} \\ r = 1: & \text{DC} \end{cases}$

2. Sinusoidal

CT	DT
$x(t) = A\cos(\omega t + \phi)$	$x[n] = A\cos(\Omega n + \phi)$

Elementary signals (cont.)



3. Euler's identity

$$\boxed{e^{j\pi} = -1}$$

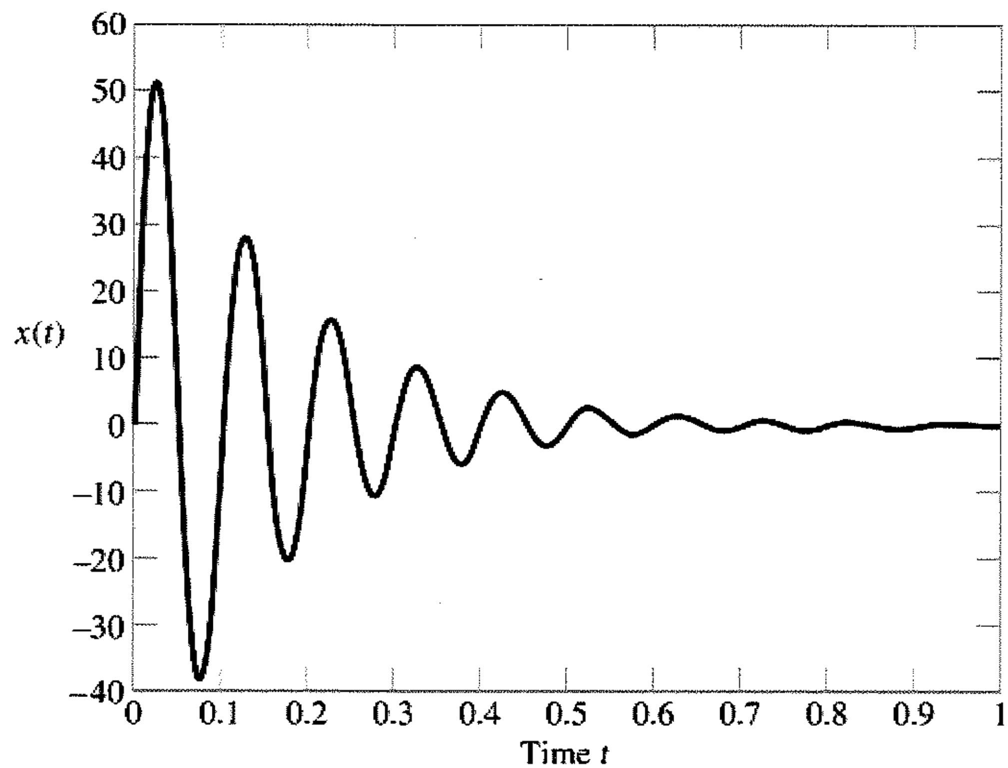
$$
\begin{aligned}
e^{j\theta} &= \cos(\theta) + j\sin\theta. \text{ Let } B = Ae^{j\phi} \\
Be^{j\omega t} &= Ae^{j\phi}e^{j\omega t} = Ae^{j(\omega t+\phi)} \\
&= A\cos(\omega t+\phi) + jA\sin(\omega t+\phi)
\end{aligned}
$$

$$
\begin{cases}
A\cos(\omega t+\phi) = \Re\{Be^{j\omega t}\} \\
A\sin(\omega t+\phi) = \Im\{Be^{j\omega t}\}
\end{cases}
$$

Elementary signals (cont.)

4. Exponentially damped sinusoidal

$$
\begin{aligned}
x(t) &= Ae^{-\alpha t}\sin(\omega t + \phi),\ \alpha > 0 \text{ for damped} \\
x[n] &= Br^{n}\sin(\Omega n + \phi),\ 0 < r < 1 \text{ for damped}
\end{aligned}
$$



Elementary signals (cont.)

Note:

- $x[n]$ May or may not be periodic
- If $\Omega N = 2\pi m$, m integer, or $\Omega = \frac{2\pi m}{N}$ (rads/sample), then $x[n]$ periodic: $x[n+N] = x[n]$
- Ω (unit?) rads/sample; N samples; ΩN radians (or simply radians if n is designated as dimensionless).

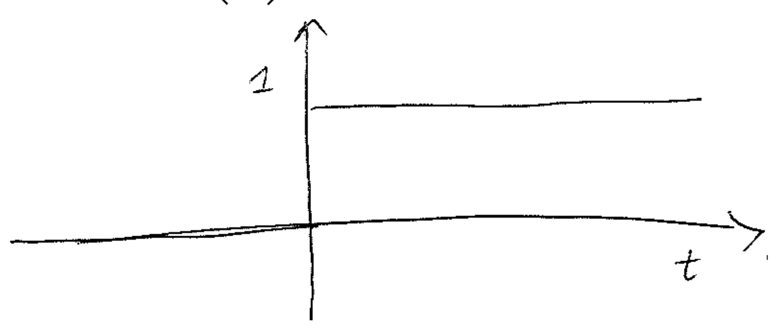
E: $x_1[n] = \sin\left(\frac{2\pi}{21}n\right)$, $x_2[n] = \sqrt{3}\cos\left(\frac{4\pi}{7}n\right)$. Fundamental period of $y[n] = x_1[n] + x_2[n]$?

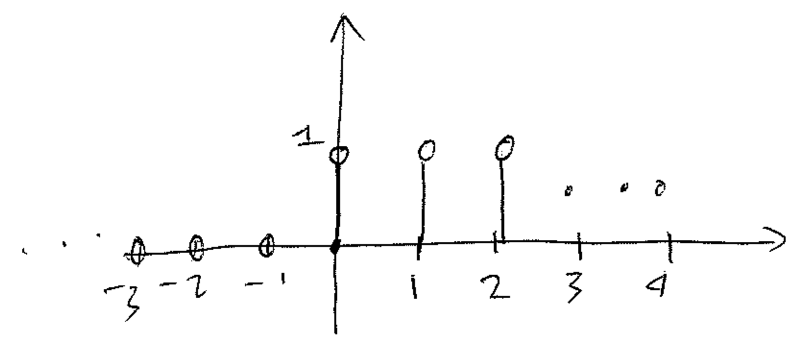
Elementary signals (cont.)

5. Step function

CT	DT
$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$	$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$

Note: $u(0)$ is not defined $u[0] = 1$.

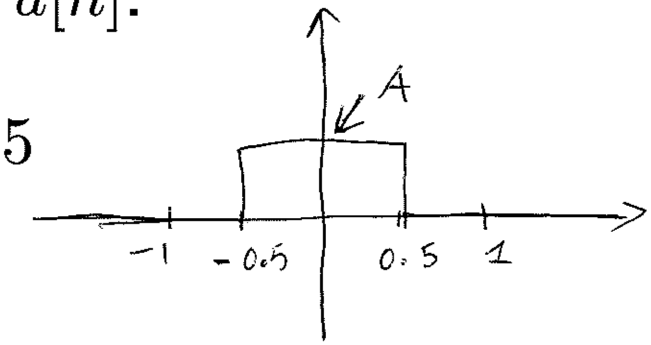




Elementary signals (cont.)

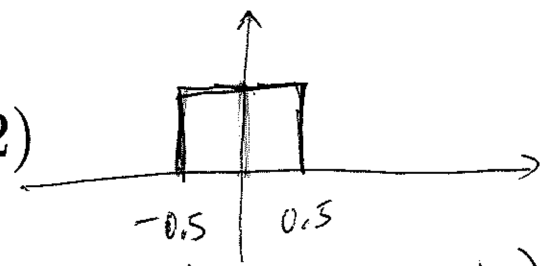
E: Rectangular pulses in terms of $u(t)$ and $u[n]$.

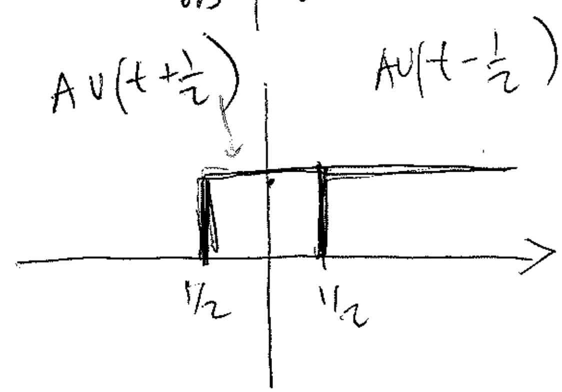
$$x(t) = \begin{cases} A, & 0 \le |t| \le 0.5 \\ 0, & |t| > 0.5 \end{cases}$$



$x(t)$ can be expressed in terms of $u(t)$ as

$$x(t) = Au(t+1/2) - Au(t-1/2)$$

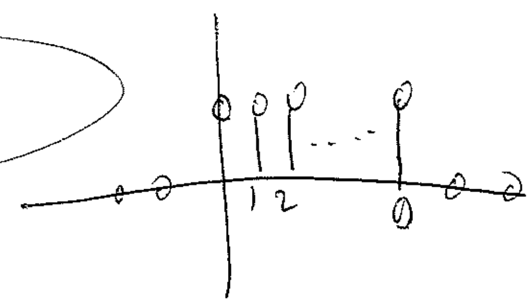




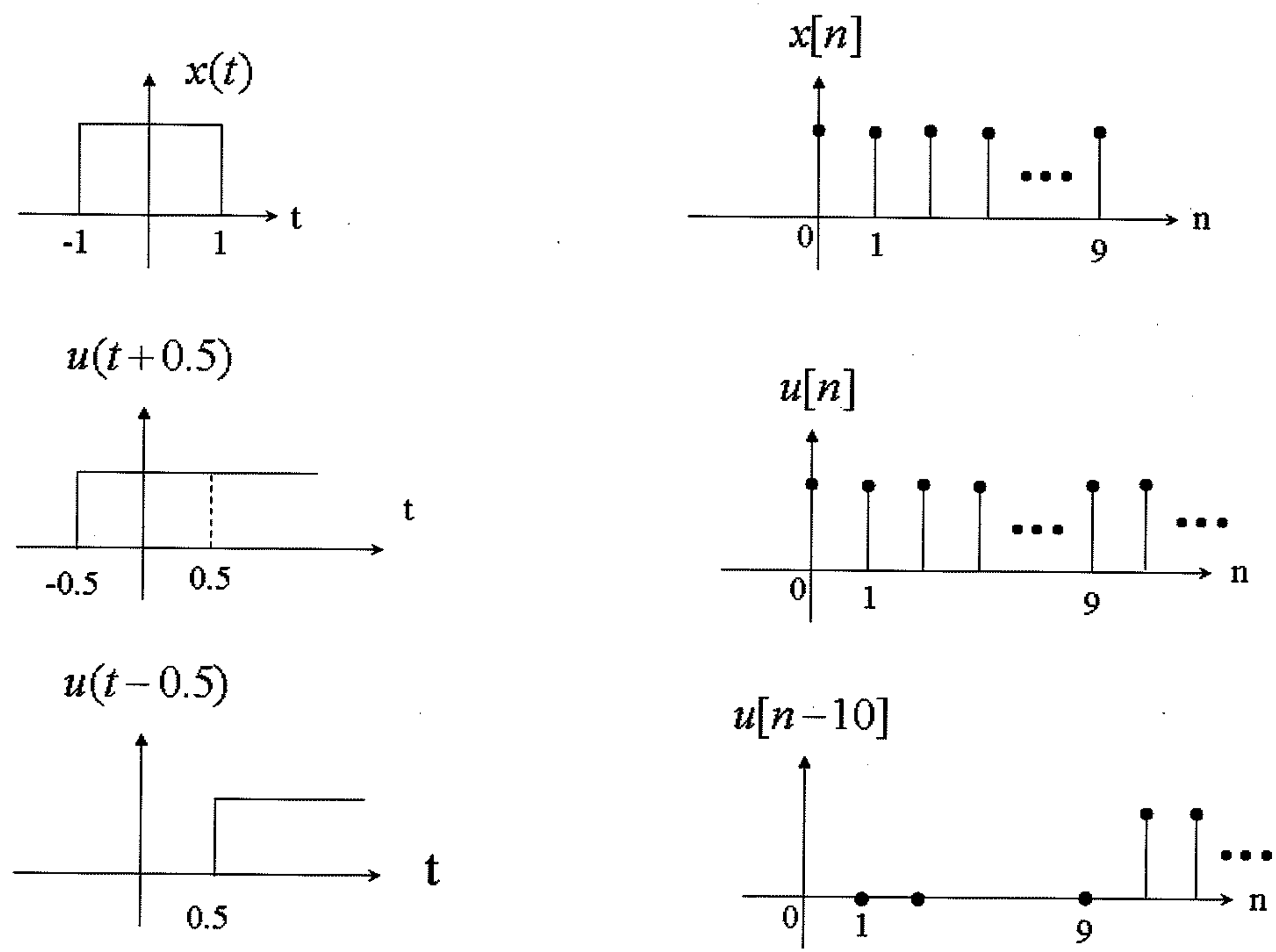
$$x[n] = \begin{cases} 1, & 0 \le n \le 9 \\ 0, & o.w. \end{cases}$$

$x[n]$ can be expressed in terms of $u[n]$ as

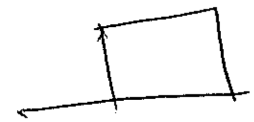
$$x[n] = u[n] - u[n-10]$$



Elementary signals (cont.)



Elementary signals (cont.)



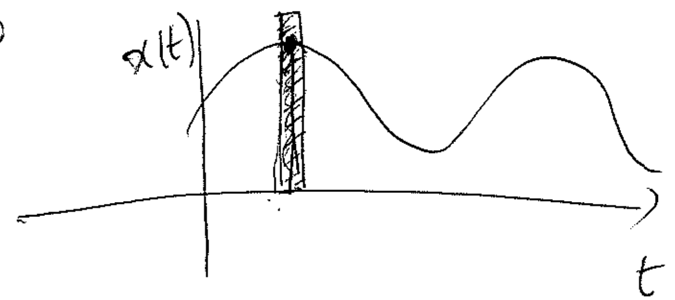
6. Impulse function

CT	DT
$\delta(t) = \begin{cases} 0, \ t \neq 0 \\ \int_{-\infty}^{\infty} \delta(t)dt = \int_{0^-}^{0^+} \delta(t)dt = 1 \end{cases}$	$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$

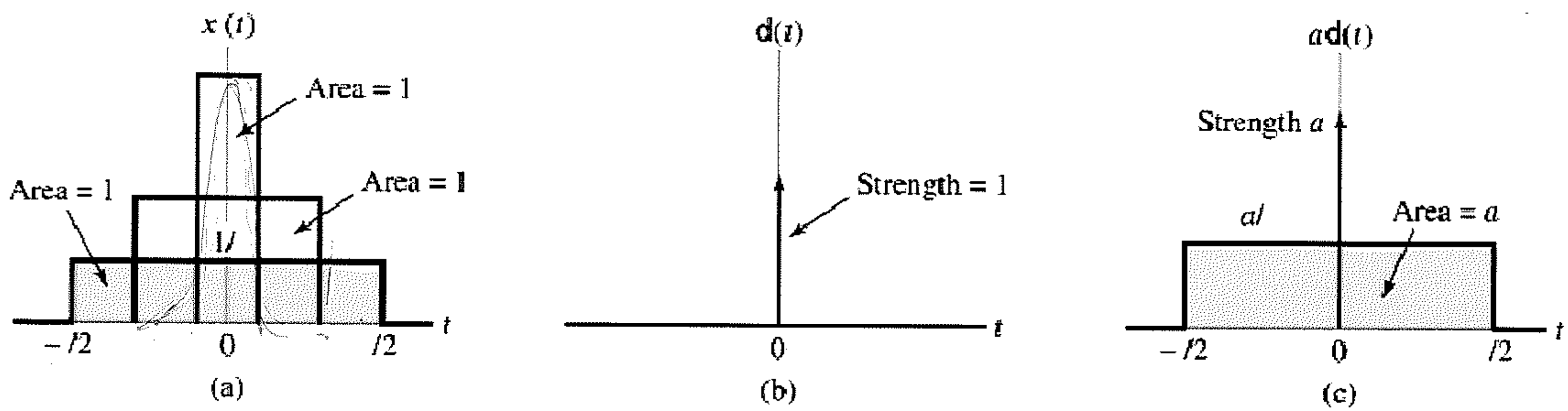
= height

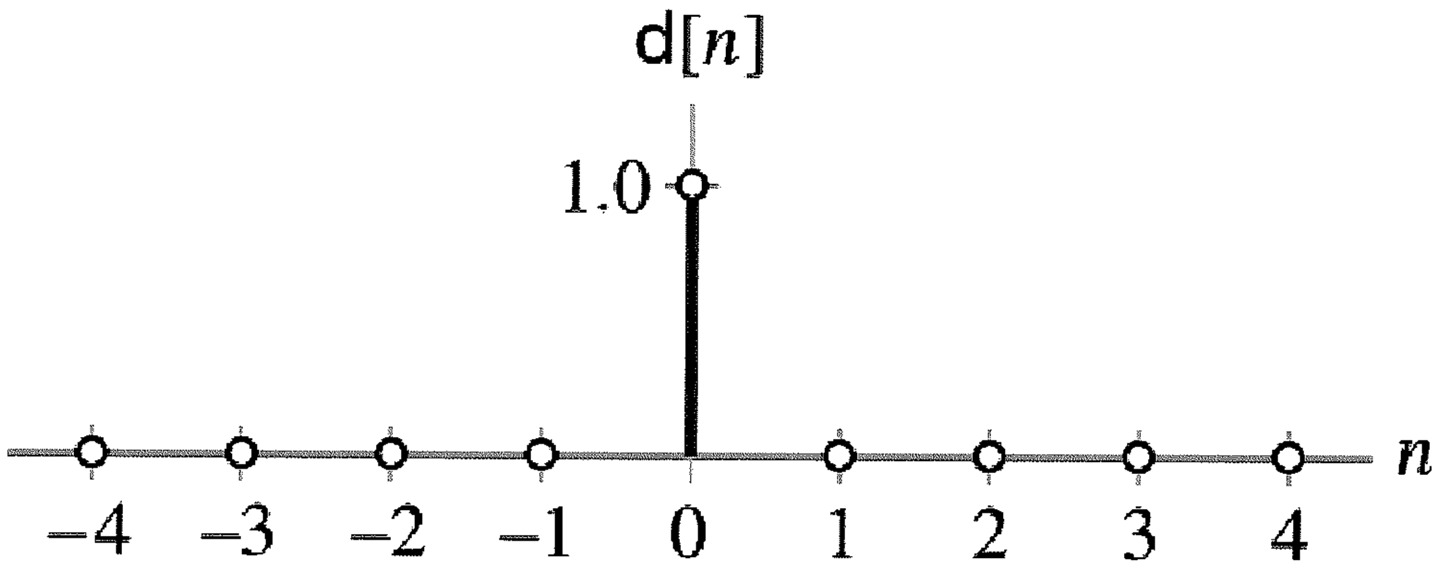
- Let $x_\Delta(t)$ be a rectangle with area one and width Δ and height $1/\Delta$. Then, $\delta(t) = \lim_{\Delta \to 0} \cancel{\Delta} x_\Delta(t)$
- $\delta(-t) = \delta(t)$ ✓
- $\int_{-\infty}^{\infty} x(t)\delta(t-t_0)dt = x(t_0)$ ✓ sift
- $\delta(t) = \frac{d}{dt}u(t) \Rightarrow u(t) = \int_{-\infty}^{t} \delta(\tau)d\tau$
- $\delta(at) = \frac{1}{|a|}\delta(t), \ \cancel{a > 0}$
- $\int_{-\infty}^{\infty} x(t)\frac{d}{dt}\delta(t-t_0)dt = \frac{d}{dt}x(t)|_{t=t_0}$
- $\int_{-\infty}^{\infty} x(t)\frac{d^n}{dt^n}\delta(t-t_0)dt = \frac{d^n}{dt^n}x(t)|_{t=t_0}$

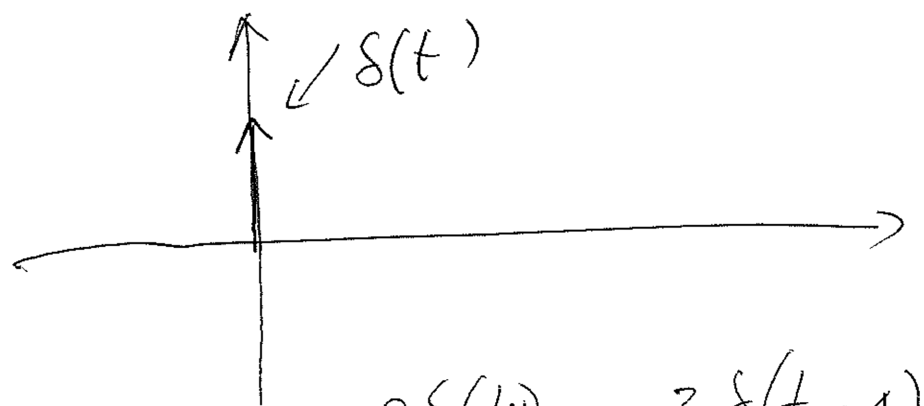
$\int_{-\infty}^{\infty} e^{-6t}\delta(t-2)dt = e^{-12}$



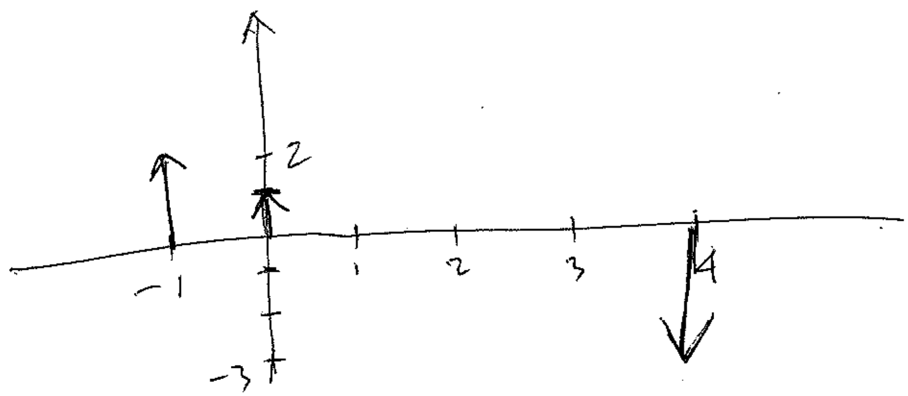
Elementary signals (cont.)







$$x(t) = 2\delta(t+1) - 3\delta(t-4) + 3\underbrace{\delta(3t)}_{=\frac{1}{3}\delta(t)} \qquad 3\left(\tfrac{1}{3}\delta(t)\right) = \delta(t)$$



Elementary signals (cont.)

7. Unit ramp function

$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases} = \int_{-\infty}^{t} u(\tau)d\tau = \int_{0}^{t} 1d\tau = tu(t)$$

$$r[n] = \begin{cases} n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$