Example

homogenous equation

$$y[n] - \frac{1}{4} y[n-1] = 0 \qquad y^{(h)}[n] = c_1 \left(\frac{1}{4}\right)^n$$

$$y[n] - \frac{1}{4} y[n-1] = \left(\frac{1}{2}\right)^n = x[n]$$

Assume a particular solution of the form

$$y^{(p)}[n] = c_p \left(\frac{1}{2}\right)^n$$

(for input in the form of $\alpha^n u[n]$)

1) Subtituting $y^{(p)}$ into the diffence equation.

$$c_p \left(\frac{1}{2}\right)^n - \frac{1}{4} c_p \left(\frac{1}{2}\right)^{n-1} = \left(\frac{1}{2}\right)^n$$

dividing both sides by $\left(\frac{1}{2}\right)^n$

$$(c_p) - \left(\frac{1}{4} c_p\right)(2) = 1$$

$$\Rightarrow \quad \frac{1}{2} c_p = 1 \quad \Rightarrow \quad c_p = 2$$

$$y^{(p)} = 2 \left(\frac{1}{2}\right)^n$$

ex 2: Find the particular solution for

$$RC\frac{dy(t)}{dt} + y(t) = x(t), \quad x(t) = \cos w_0 t$$

guess: $y^{(p)}(t) = c_1 \cos w_0 t + c_2 \sin w_0 t$.

$$RC\left(-c_1 w_0 \sin w_0 t + c_2 w_0 \cos w_0 t\right) + c_1 \cos w_0 t + c_2 \sin w_0 t = \cos w_0 t$$

(by substitution)

$$\begin{cases} \cos w_0 t \left(RC c_2 w_0 + c_1 \right) = \cos w_0 t \\ \sin w_0 t \left(-RC c_1 w_0 + c_2 \right) = 0 \end{cases}$$

$$\begin{cases} RC c_2 w_0 + c_1 = 1 & (1) \\ -RC c_1 w_0 + c_2 = 0 & (2) \end{cases}$$

multiply (1) by $+RCw_0$ then add the result to (2)

$$\Rightarrow \begin{aligned} (RCw_0)^2 c_2 + c_1 RCw_0 &= RCw_0 \\ + \quad -RC c_1 w_0 + c_2 &= 0 \end{aligned}$$

$$\Rightarrow c_2 = \frac{RCw_0}{(RCw_0^2 + 1)}, \quad c_1 = \frac{1}{1 + (RCw_0)^2}$$

$$y^{(p)}(t) = \left(\frac{1}{1+(RC\omega_0)^2}\right)\cos\omega_0 t + \frac{RC\omega_0}{(RC\omega_0^2+1)}\sin\omega_0 t$$

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ex: $y[n] - \frac{1}{4} y[n-1] = \left(\frac{1}{2}\right)^n u[n] \quad n \geq 0 \quad (\Delta)$ ④

$y[-1] = 8$

1). Find the homogenous solution $y^{(h)}$

2) Find particular solution $y^{(p)}$

3) Find the coefients in the homogenous solution using initial conditions

the general solution $y(t) = y^{(h)}(t) + y^{(p)}(t)$ (*)

$y[n] = y^{(h)}[n] + y^{(p)}[n]$

Note: In step 3) if the input $x[n]$ is of the form $x[n]u[n]$, then we first must translate initial conditions to the time $n = 0$.

This is because the input is valid for $n \geq 0$ only.

1) $y^{(h)}[n] = c\left(\frac{1}{4}\right)^n$

2) $y^{(p)}[n] = 2\left(\frac{1}{2}\right)^n$ (See previous example)

3) $y[0] - \frac{1}{4}y[-1] = 1$ (using the difference equation Δ)

$y[0] = 1 + 2 = 3$

Now, since we have

$y[n] = c\left(\frac{1}{4}\right)^n + 2\left(\frac{1}{2}\right)^n$ (using $\#$)

$y[0] = c\left(\frac{1}{4}\right)^0 + 2\left(\frac{1}{2}\right)^0$

$3 = c + 2 \Rightarrow c = 1$

$y[n] = \left[\left(\frac{1}{4}\right)^n + 2\left(\frac{1}{2}\right)^n\right] u[n]$