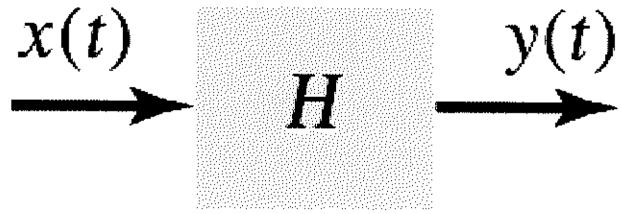
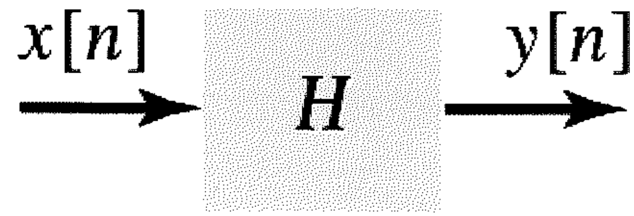
System properties/characteristics



(a)



(b)

Notation: Let $\mathcal{H}$ represent the system, $x(t) \xrightarrow{\mathcal{H}} y(t)$ represent a system with input $x(t)$ and output $y(t)$.

1. Stability: BIBO (bounded input $\Rightarrow$ bounded output) stability

Assume (CT) $|x(t)| \leq M_x < \infty, \ \forall t \Rightarrow |y(t)| \leq M_y < \infty, \ \forall t$

(DT) $|x[n]| \leq M_x < \infty, \ \forall n \Rightarrow |y[n]| \leq M_y < \infty \ \forall n$

System properties/characteristics (cont.)

E: Is the system with $y(t) = e^{at}x(t)$ for $a > 0$ stable (BIBO)?

$$|y(t)| = |e^{at}x(t)| = e^{at}|x(t)| \leq e^{at} M_x \quad (\text{because } |x(t)| \leq M_x)$$

since $e^{at} M_x$ is not finite $\forall t \Rightarrow$ the system is BIBO unstable.

$$\boxed{\begin{array}{l} |ab| \leq |a||b| \\ |a+b| \leq |a| + |b| \end{array}}$$

$$y[n] = \sum_{k=0}^{\infty} \rho^k x[n-k]$$

$$|y[n]| = \left|\sum_{k=0}^{\infty} \rho^k x[n-k]\right| \leq \sum_{k=0}^{\infty} |\rho^k x[n-k]| \leq \sum_{k=0}^{\infty} |\rho^k||x[n-k]| \leq \sum_{k=0}^{\infty} |\rho^k| M_x$$

$$= M_x \sum_{k=0}^{\infty} |\rho^k| \begin{cases} = \infty & \text{if } |\rho| \geq 1 \\ < \infty & \text{if } |\rho| < 1 \end{cases}$$

System properties/characteristics (cont.)

2. Memory/memoryless

- Memory system: present output value depends on future/past input.
- Memoryless system: present output value depends only on present input.

E: Memory systems:

t: current
$t-1$: past
$t+1$: future

$$y(t) = 5x(t) + \int_{-\infty}^{t} x(\tau)d\tau$$

causal

$$y[n] = \sum_{m=n-5}^{n+5} x[m] = x[n-5] + x[n-4] + x[n-3] + \cdots x[n+4] + x[n+5]$$

non-causal

Memoryless systems:

$$y[n] = x[n] + x^2[n]$$

System properties/characteristics (cont.)

3. Causal/noncausal

   - Causal: present output depends on present/past values of input.
   - Noncausal: present output depends on future values of input.

   Note: Memoryless $\Rightarrow$ causal, but causal not necessarily be memoryless.

4. Time invariance (TI): time delay or advance of input $\Rightarrow$ an identical time shift in the output.
   Let us defin a system mapping $y(t) = \mathcal{H}(x(t))$. The system is time-invariant if

$$x(t - t_0) \xrightarrow{\mathcal{H}} y(t - t_0)$$

$$x[n - n_0] \xrightarrow{\mathcal{H}} y[n - n_0]$$