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System properties/characteristics (cont.)

3. Causal/noncausal

   - Causal: present output depends on present/past values of input.
   - Noncausal: present output depends on future values of input.

   Note: Memoryless $\Rightarrow$ causal, but causal not necessarily be memoryless.

4. Time invariance (TI): time delay or advance of input $\Rightarrow$ an identical time shift in the output.
   Let us define a system mapping $y(t) = \mathcal{H}(x(t))$. The system is time-invariant if

$$x'(t) = x(t - t_0) \xrightarrow{\mathcal{H}} y(t - t_0) = y'(t)$$

$$x[n - n_0] \xrightarrow{\mathcal{H}} y[n - n_0]$$

System properties/characteristics (cont.)

E: Is system $y[n] = r^n x[n]$ time invariant?

$x'[n] = x[n-n_0]$

$y'[n] = H(x'[n]) = r^n x'[n]$ (replacing $x[n]$ with $x'[n]$)

$= r^n x[n-n_0]$

check to see if $y'[n] \neq y[n-n_0] \Rightarrow TV$ $y[n-n_0] = r^{n-n_0} x[n-n_0]$ (replacing n with $n-n_0$)

- $y(t) = e^{at}x^2(t)$:

$x'(t) = x(t-t_0)$, $y'(t) = e^{at}[x'(t)]^2 = e^{at}[x(t-t_0)]^2$

$y(t-t_0) = e^{a(t-t_0)}[x(t-t_0)]^2$; $y'(t) \neq y(t-t_0) \Rightarrow TV$

- $y[n] = u[n]x[n]$:

$x'[n] = x[n-n_0]$, $y'[n] = u[n]x'[n] = u[n]x[n-n_0]$

$y[n-n_0] = u[n-n_0]x[n-n_0] \Rightarrow y[n-n_0] \neq y'[n] \Rightarrow TV$

(3)

- $y(t) = x^2(t)$

$x'(t) = x(t-t_0)$; $y'(t) = (x'(t))^2 = (x(t-t_0))^2 = x^2(t-t_0)$

$y(t-t_0) = x^2(t-t_0) \Rightarrow y'(t) = y(t-t_0) \Rightarrow TI$

- $y[n] = \frac{x[n] + x[n+1]}{2}$

$x'[n] = x[n-n_0]$, $y'[n] = \frac{x'[n] + x'[n+1]}{2} = \frac{x[n-n_0] + x[n-n_0+1]}{2}$

$y[n-n_0] = \frac{x[n-n_0] + x[n-n_0+1]}{2}$

$y'[n] = y[n-n_0] \Rightarrow TI$

System properties/characteristics (cont.)

5. Linearity

   Linear system: If $x_1(t) \xrightarrow{\mathcal{H}} y_1(t)$, $x_2(t) \xrightarrow{\mathcal{H}} y_2(t)$, then $ax_1(t) + bx_2(t) \xrightarrow{\mathcal{H}} ay_1(t) + by_2(t)$. Else, nonlinear.

   - Superposition property (addition)
   - Homogeneity (scaling)

   The following operations preserve linearity

   - $\frac{dx(t)}{dt} \xrightarrow{\mathcal{H}} \frac{dy(t)}{dt}$
   - $\int_{-\infty}^{t} x(\tau)d\tau \xrightarrow{\mathcal{H}} \int_{-\infty}^{t} y(\tau)d\tau$
   - $\sum_{m=-\infty}^{n} x[m] \xrightarrow{\mathcal{H}} \sum_{m=-\infty}^{n} y[m]$

System properties/characteristics (cont.)

E:

$$x_1[n] \xrightarrow{H} y_1[n]$$
$$x_2[n] \xrightarrow{H} y_2[n]$$

- $y[n] = nx[n-3]$: linear

$x'[n] = a x_1[n] + b x_2[n]$, $y'[n] = n x'[n-3] = n\left(a x_1[n-3] + b x_2[n-3]\right)$

$$= a\, \underbrace{n x_1[n-3]} + b\, \underbrace{n x_2[n-3]}$$
$$= a\, y_1[n] + b\, y_2[n]$$

$y'[n] = a y_1[n] + b y_2[n] \Rightarrow$ Linear

- $y(t) = 5x(t+t_0)$: linear

$x_1(t) \xrightarrow{H} y_1(t) = 5x_1(t+t_0)$
$x_2(t) \xrightarrow{H} y_2(t) = 5x_2(t+t_0)$

$x'(t) = a x_1(t) + b x_2(t)$ $5x'(t+t_0) = y'(t) = 5\left(a x_1(t+t_0) + b x_2(t+t_0)\right)$

$$= a \underbrace{5x_1(t+t_0)}_{= y_1(t)} + b \underbrace{5x_2(t+t_0)}_{= y_2(t)}$$

$\Rightarrow$ Linear

- $y(t) = |x(t)|$: nonlinear

$x_1(t) \xrightarrow{H} y_1(t) = |x_1(t)|$
$x_2(t) \xrightarrow{H} y_2(t) = |x_2(t)|$
$x'(t) = x_1(t) + x_2(t)$ $y'(t) = |x'(t)| = |x_1(t) + x_2(t)| \overset{?}{\neq} |x_1(t)| + |x_2(t)|$

$\Rightarrow$ non-linear

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Time-Domain Representation of LTISystems

$$x(t) \longrightarrow \boxed{H} \longrightarrow y(t)$$

(a)

$$x[n] \longrightarrow \boxed{H} \longrightarrow y[n]$$

(b)

- System $\mathcal{H}$ is a linear time-invariant (LTI) system.
- How to analyze a system. Given an input, find system output.
- Impulse response of an LTI system $\mathcal{H}$:

Convolution sum

$$
\begin{aligned}
x(t) &= \delta(t) \xrightarrow{\mathcal{H}} y(t) = h(t) \\
x[n] &= \delta[n] \xrightarrow{\mathcal{H}} y[n] = h[n]
\end{aligned}
$$

where $h(t)$ (CT) and $h[n]$ (DT) are the system impulse responses.

$h(t)$ or $h[n]$ completely characterizes an LTI system.
By knowing $h(t)$ or $h[n]$, system output can be obtained for an arbitrary input signal $x(t)$ or $x[n]$.
How is $y(t)/y[n]$ related to $x(t)/x[n]$ and $h(t)/h[n]$?