

System properties/characteristics (cont.)

3. Causal/noncausal

- Causal: present output depends on present/past values of input.
- Noncausal: present output depends on future values of input.

Note: Memoryless \Rightarrow causal, but causal not necessarily be memoryless.

4. Time invariance (TI): time delay or advance of input \Rightarrow an identical time shift in the output.

Let us define a system mapping $y(t) = \mathcal{H}(x(t))$. The system is time-invariant if

$$x'(t) =: x(t - t_0) \xrightarrow{\mathcal{H}} y(t - t_0) = y'(t)$$

$$x[n - n_0] \xrightarrow{\mathcal{H}} y[n - n_0]$$

System properties/characteristics (cont.)

E: Is system $y[n] = r^n x[n]$ time invariant?

$$\begin{aligned} x'[n] &= x[n-n_0] \\ y'[n] &= H(x'[n]) = r^n x'[n] \quad (\text{replacing } x[n] \text{ with } x'[n]) \\ &= r^n x[n-n_0] \end{aligned}$$

check to see if $y'[n] \stackrel{?}{=} y[n-n_0] \Rightarrow \text{TV}$ $y[n-n_0] = r^{n-n_0} x[n-n_0]$ (replacing n with $n-n_0$)

• $y(t) = e^{at} x^2(t)$:

$$\begin{aligned} x'(t) &= x(t-t_0), \quad y'(t) = e^{at} [x'(t)]^2 = e^{at} [x(t-t_0)]^2 \\ y(t-t_0) &= e^{a(t-t_0)} [x(t-t_0)]^2; \quad y'(t) \neq y(t-t_0) \Rightarrow \text{TV} \end{aligned}$$

• $y[n] = u[n]x[n]$:

$$\begin{aligned} x'[n] &= x[n-n_0], \quad y'[n] = u[n]x'[n] = u[n]x[n-n_0] \\ y[n-n_0] &= u[n-n_0]x[n-n_0] \Rightarrow y[n] \neq y'[n] \Rightarrow \text{TV} \end{aligned}$$

$$y(t) = x^2(t)$$

$$x'(t) = x(t-t_0)$$

$$y(t-t_0) = x^2(t-t_0)$$

$$y'(t) = x'(t)$$

$$\Rightarrow y'(t) = y(t-t_0) \Rightarrow \text{TI}$$

$$x^2(t) = (x'(t))^2$$

$$x^2(t-t_0) = (x(t-t_0))^2$$

(3)

$$x^2(t) = x^2(t-t_0)$$

$$y[n] = \frac{x[n] + x[n+1]}{2}$$

$$x'[n] = x[n-n_0], \quad y'[n] = \frac{x'[n] + x'[n+1]}{2}$$

$$y[n-n_0] = \frac{x[n-n_0] + x[n-n_0+1]}{2}$$

$$y'[n] = y[n-n_0] \Rightarrow \text{TI}$$

$$x[n-n_0] + x[n-n_0+1]$$

System properties/characteristics (cont.)

5. Linearity

Linear system: If $x_1(t) \xrightarrow{\mathcal{H}} y_1(t)$, $x_2(t) \xrightarrow{\mathcal{H}} y_2(t)$, then $ax_1(t) + bx_2(t) \xrightarrow{\mathcal{H}} ay_1(t) + by_2(t)$. Else, nonlinear.

- Superposition property (addition)
- Homogeneity (scaling)

The following operations preserve linearity

- $\frac{dx(t)}{dt} \xrightarrow{\mathcal{H}} \frac{dy(t)}{dt}$
- $\int_{-\infty}^t x(\tau) d\tau \xrightarrow{\mathcal{H}} \int_{-\infty}^t y(\tau) d\tau$
- $\sum_{m=-\infty}^n x[m] \xrightarrow{\mathcal{H}} \sum_{m=-\infty}^n y[m]$

System properties/characteristics (cont.)

E:

$$\begin{matrix} x_1[n] & \xrightarrow{H} & y_1[n] \\ x_2[n] & \xrightarrow{H} & y_2[n] \end{matrix}$$

• $y[n] = nx[n-3]$: **linear**

$$y'[n] = a x_1[n] + b x_2[n] = n (a x_1[n-3] + b x_2[n-3])$$

$$= a n x_1[n-3] + b n x_2[n-3]$$

$$= a y_1[n] + b y_2[n]$$

\Rightarrow Linear

$$y'[n] =$$

• $y(t) = 5x(t + t_0)$: **linear**

$$x_1(t) \xrightarrow{H} y_1(t) = 5x_1(t+t_0)$$

$$x_2(t) \xrightarrow{H} y_2(t) = 5x_2(t+t_0)$$

$$x'(t) = a x_1(t) + b x_2(t)$$

$$5x'(t+t_0) = y'(t) = 5(a x_1(t+t_0) + b x_2(t+t_0))$$

$$= a 5x_1(t+t_0) + b 5x_2(t+t_0)$$

$$\stackrel{H}{=} y_1'(t) + y_2'(t)$$

\Rightarrow Linear

• $y(t) = |x(t)|$: **nonlinear**

$$x_1(t) \xrightarrow{H} y_1(t) = |x_1(t)|$$

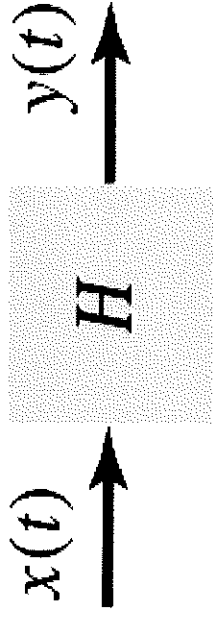
$$x_2(t) \xrightarrow{H} y_2(t) = |x_2(t)|$$

$$x'(t) = x_1(t) + x_2(t) \xrightarrow{H} y'(t) = |x_1(t) + x_2(t)|$$

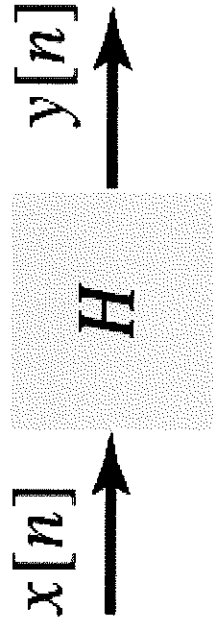
$$\neq |x_1(t)| + |x_2(t)|$$

\Rightarrow non-linear

Time-Domain Representation of LTI Systems



(a)



(b)

- System \mathcal{H} is a linear time-invariant (LTI) system.
- How to analyze a system. Given an input, find system output.
- Impulse response of an LTI system \mathcal{H} :

Convolution sum

$$x(t) = \delta(t) \xrightarrow{\mathcal{H}} y(t) = h(t)$$

$$x[n] = \delta[n] \xrightarrow{\mathcal{H}} y[n] = h[n]$$

where $h(t)$ (CT) and $h[n]$ (DT) are the system impulse responses.

$h(t)$ or $h[n]$ completely characterizes an LTI system. ■

By knowing $h(t)$ or $h[n]$, system output can be obtained for an arbitrary input signal $x(t)$ or $x[n]$. ■

How is $y(t)/y[n]$ related to $x(t)/x[n]$ and $h(t)/h[n]$?