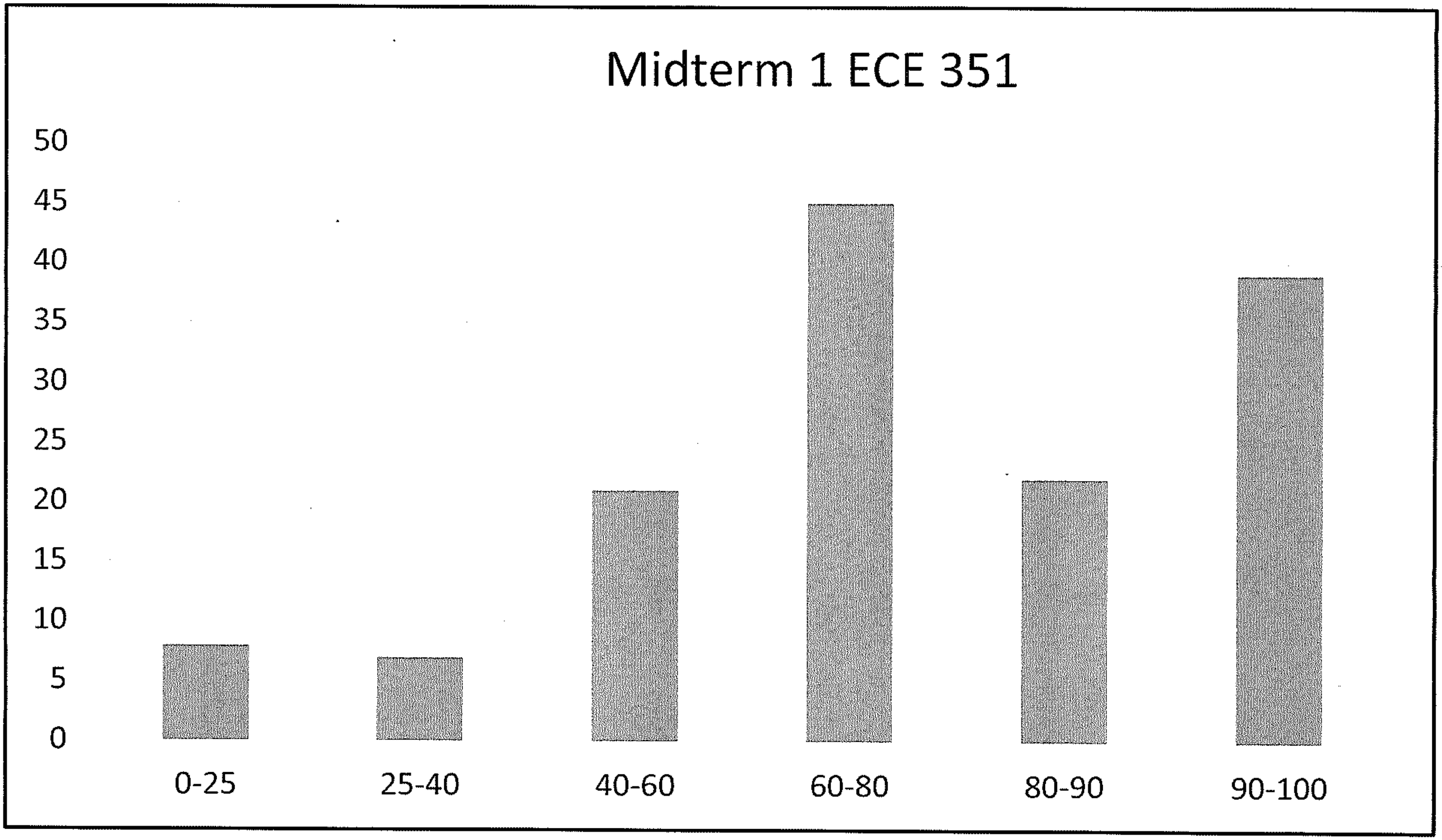
Average 72
Max 100 (10)
Min 19



Review:

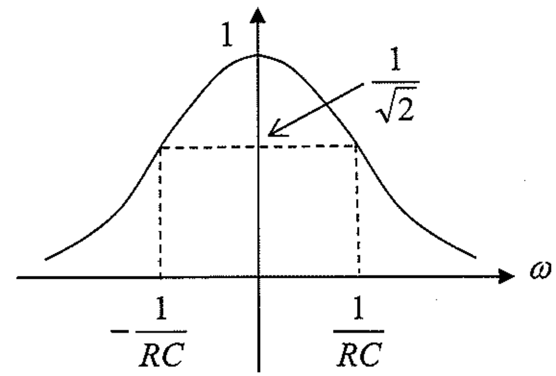
$$\begin{matrix} e^{j\Omega n} \\ e^{j\omega t} \end{matrix} \longrightarrow \underset{\text{LTI}}{\boxed{H}} \longrightarrow \begin{matrix} H(e^{j\Omega})\, e^{j\Omega n} \\ H(j\omega)\, e^{j\omega t} \end{matrix}$$

Sa

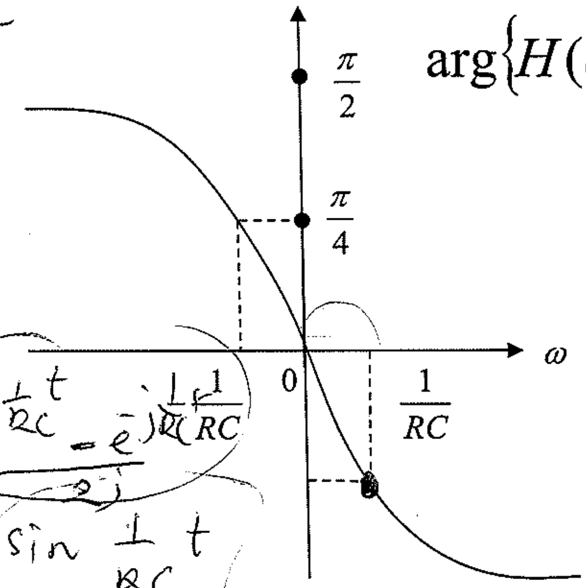
$5V$ ——— DC

(2)

$|H(j\omega)|$



$\arg\{H(e^{j\Omega})\} = \underline{/H(j\omega)}$



$x(t) = \sin \frac{1}{RC} t$ $\quad \frac{e^{j\frac{1}{RC}t} - e^{-j\frac{1}{RC}t}}{2j}$

$e^{j\omega t}$: eigenfunction of the LTI system (eigen value $\lambda = H(j\omega)$)

$(H\{e^{j\omega t}\} = \lambda e^{j\omega t})$

Now, if the input to an LTI system is expressed as a weighted sum of *M* complex sinusoids:

$$\boxed{x(t) = \sum_{k=1}^{M} a_k e^{j\omega_k t}, \text{ then} \qquad y(t) = \sum_{k=1}^{M} a_k H(j\omega_k) e^{j\omega_k t}}$$

$M \left(e^{j\omega_k t} \xrightarrow{H} H(j\omega_k) e^{j\omega_k t} \right)$

$\sum_{k=1}^{M} a_k e^{j\omega_k t} \xrightarrow{H} \sum_{k=1}^{M} a_k H(j\omega_k) e^{j\omega_k t}$

ex: $x(t) = 2\cos w_1 t - \cos w_2 t$ $\qquad H(jw) = \dfrac{1}{RC\left(\frac{1}{RC} + jw\right)}$

$y(t) = ?$

Using Euler's formula:

$$\boxed{\begin{aligned} \cos\theta &= \frac{e^{j\theta} + e^{-j\theta}}{2} \\ \sin\theta &= \frac{e^{j\theta} - e^{-j\theta}}{2j} \end{aligned}}$$

$x(t) =$

$$2\left(\frac{e^{jw_1 t} + e^{-jw_1 t}}{2}\right) - \cancel{\cos}\left(\frac{e^{jw_2 t} + e^{-jw_2 t}}{2}\right)$$

$$\cancel{x}(t) = e^{-jw_1 t} + e^{jw_1 t} - \frac{1}{2}e^{jw_2 t} - \frac{1}{2}e^{-jw_2 t}, \quad \text{let } w_1 = 2,\ w_2 = 3$$

$$\boxed{\cancel{x}(t) = \sum_{k=1}^{M} a_k e^{jw_k t}} = \boxed{e^{-j2t} + e^{j2t} - \frac{1}{2}e^{j3t} - \frac{1}{2}e^{-j3t}}$$

compare

$a_1 = 1, \quad w_1 = -2$

$a_2 = 1, \quad w_2 = 2$

$a_3 = -\frac{1}{2}, \quad w_3 = 3$

$a_4 = -\frac{1}{2} \quad w_4 = -3$

$$y(t) = \sum_{k=1}^{M} H(jw_k)\, a_k e^{jw_k t}$$

$$= \frac{1}{RC\left(\frac{1}{RC} + j(-2)\right)}(1)\, e^{-j2t} + \frac{1}{RC\left(\frac{1}{RC} + 2j\right)}(1)\, e^{2jt} + \frac{1}{RC\left(\frac{1}{RC} + 3j\right)}\left(\frac{-1}{2}\right)e^{j3t} + \frac{1}{RC\left(\frac{1}{RC} + (-3j)\right)}\left(\frac{-1}{2}\right)e^{-j3t}$$