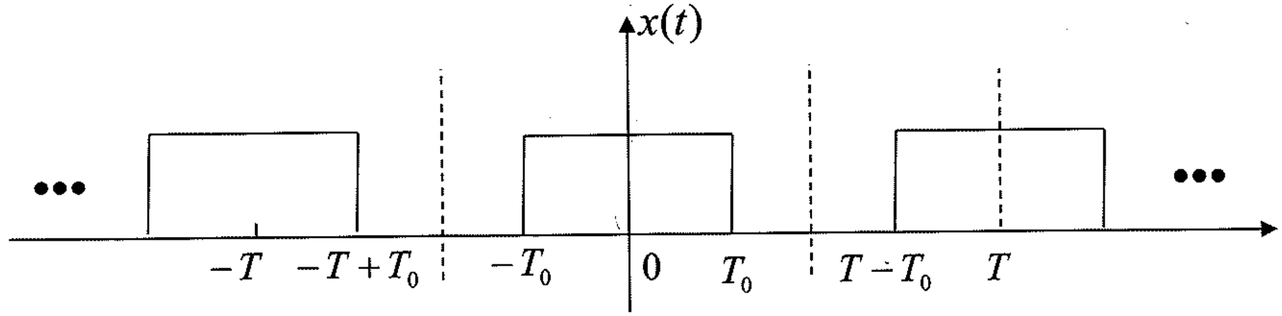
$y = f(x)$

$y' = f(ax)$ $\quad a > 1$

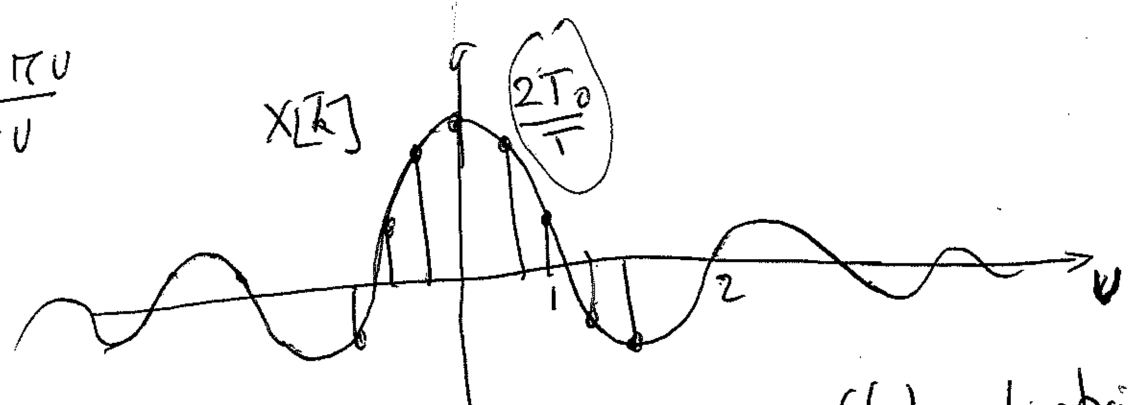
E FS of a square wave.



Period is T, so $\omega_0 = 2\pi/T$

Solution: $X[k] = \frac{2T_0}{T} \operatorname{sinc}\left(k \frac{2T_0}{T}\right)$

$\operatorname{sinc} u \triangleq \frac{\sin \pi u}{\pi u}$ $\quad u = \frac{k 2T_0}{T}$



$\frac{T_0}{T} \uparrow \rightarrow$ ~~[illegible]~~ energy of $x(t)$ distributed over smaller range of frequencies.

$\frac{T_0}{T} \downarrow \rightarrow$ 0 - w

<u>DTFT</u>   D.T., nonperiodic

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\Omega})e^{j\Omega n}\,d\Omega$$

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n]\,e^{-j\Omega n}$$

DTFT of signal x[n], also Freq-domain representation of x[n].

$$x[n] \xleftrightarrow{DTFT} X(e^{j\Omega})$$

periodic

(2π)

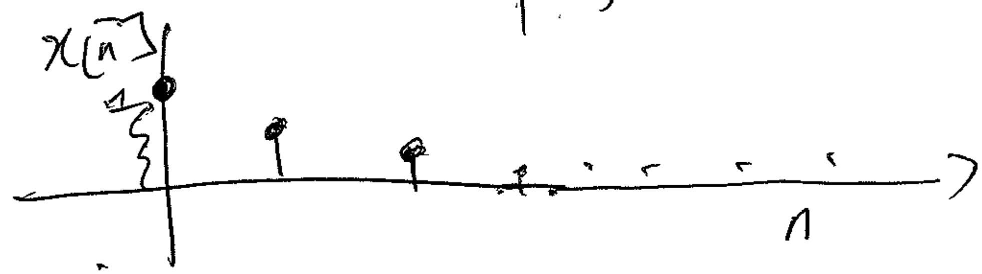
$$\boxed{X(e^{j(\Omega+2\pi)}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}\cdot \underbrace{e^{-j2\pi n}}_{=1} = X(e^{j\Omega})}$$

E

$$x[n] = \alpha^n u[n], \quad X(e^{j\Omega}) = ?$$

$|\alpha| < 1$

Solution:



Use DTFT

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n}$$

$$= \sum_{n=0}^{\infty} \alpha^n e^{-j\Omega n} = \sum_{n=0}^{\infty} \left(\alpha e^{-j\Omega}\right)^n$$

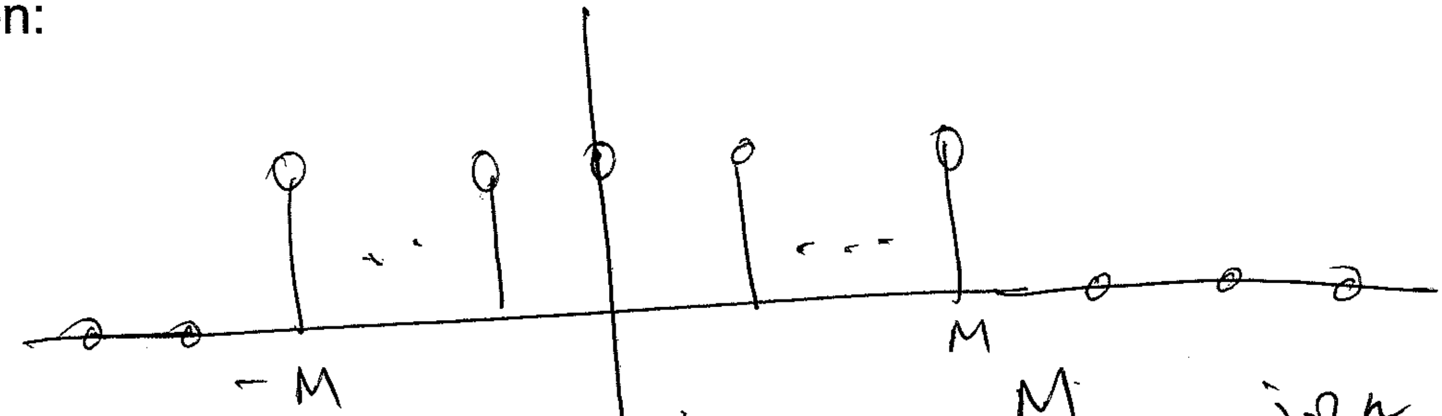
$$= \frac{1}{1 - \alpha e^{-j\Omega}}$$

E

rectangular pulse

$$x[n] = \begin{cases} 1, & |n| \le M \\ 0, & |n| > M \end{cases} \qquad X(e^{j\Omega}) = ?$$

Solution:



$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\Omega n} = \sum_{n=-M}^{M} e^{-j\Omega n}$$

$$\overset{m = n+M}{=} \sum_{m=0}^{2M} e^{-j\Omega(m-M)} = e^{j\Omega M} \sum_{m=0}^{2M} e^{-j\Omega m}$$

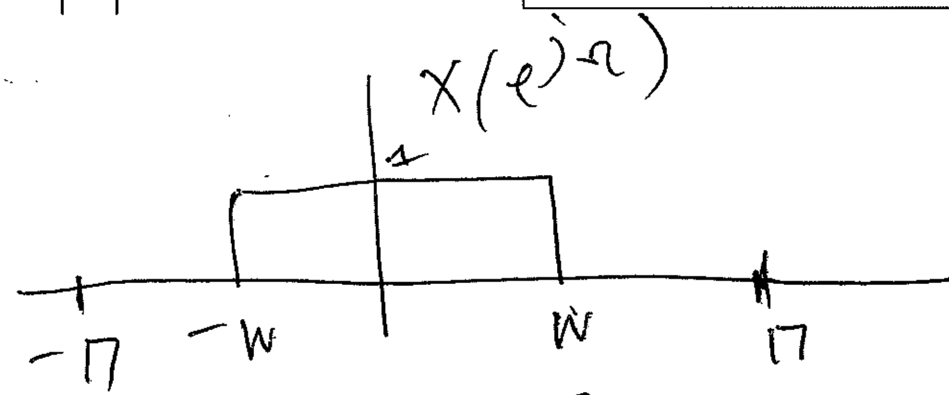
$$= e^{j\Omega M} \left(\frac{1 - e^{-j\Omega(2M+1)}}{1 - e^{-j\Omega}} \right) =$$

$\boxed{\text{E}}$ Inverse DTFT of a rectangular spectrum, Example [redacted]

$$X(e^{j\Omega}) = \begin{cases} 1, & |\Omega| < W \\ 0, & W < |\Omega| < \pi \end{cases}$$

$X(e^{j\Omega})$ is defined over $(-\pi, \pi)$ periodic in Ω

Solution:



$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\Omega}) e^{j\Omega n} \, d\Omega$$

$$= \frac{1}{2\pi} \int_{-W}^{W} (1) e^{j\Omega n} \, d\Omega = \frac{1}{2\pi} \left(\frac{e^{j\Omega n}}{jn} \right) \Bigg|_{-W}^{W}$$

$$= \frac{1}{2\pi} \left[\frac{e^{jnW} - e^{-jnW}}{jn} \right] = \frac{\sin nW}{\pi n}$$

$$= \text{sinc}\left(\frac{W}{\pi} n \right)$$