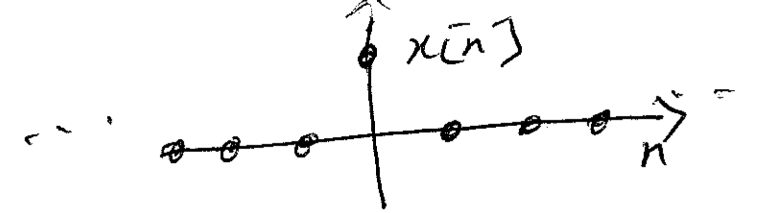
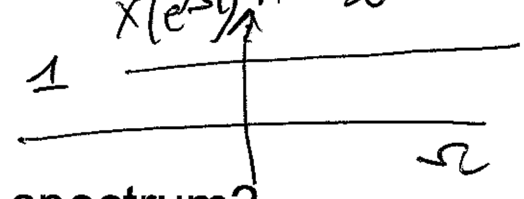
$\boxed{E}$ DTFT of unit impulse $x[n] = \delta[n]$



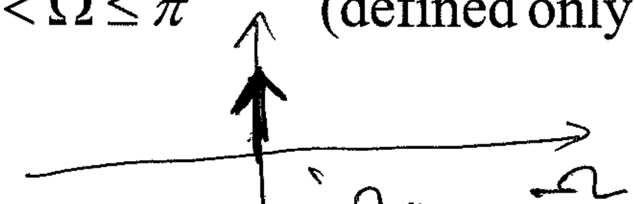
Solution: $X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n} = \sum_{n=-\infty}^{\infty} \delta[n] e^{-j\Omega n}$

$= 1$



• What about inverse DTFT of a unit impulse spectrum?

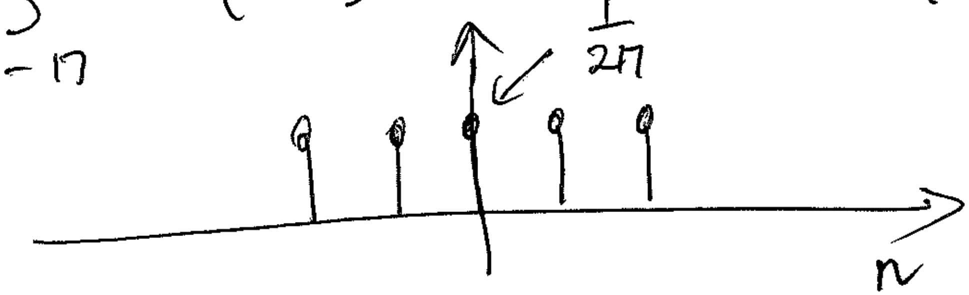
$X(e^{j\Omega}) = \delta(\Omega), \quad -\pi < \Omega \leq \pi$ (defined only one period)



Solution:

$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\Omega}) e^{j\Omega n} d\Omega$

$= \frac{1}{2\pi} \int_{-\pi}^{\pi} \delta(\Omega) e^{j\Omega n} d\Omega = \frac{1}{2\pi}$



$\boxed{\text{E}}$

$$x[n] = \begin{cases} 2^n, & 0 \le n \le 9 \\ 0, & o.w. \end{cases} \qquad X(e^{j\Omega}) = ?$$

Solution:

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\Omega n}$$

$$= \cancel{\sum_{n=0}^{9} x[n]\, e^{-j\Omega n}}$$

$$= \sum_{n=0}^{9} 2^n e^{-j\Omega n} = \sum_{n=0}^{9} \left(2 e^{-j\Omega}\right)^n$$

$$= \frac{1 - \left(2e^{-j\Omega}\right)^{9+1}}{1 - 2e^{-j\Omega}}$$

$$\left| X(e^{j\Omega}) \right| = ?$$

$X(e^{j\Omega}) = 2\cos(2\Omega)$
$x[n] = ?$

$\boxed{E}$ $X(e^{j\Omega}) = 2\cos(2\Omega)$, $x[n]=?$

Use inspection! $X(e^{j\Omega}) = e^{j2\Omega} + e^{-j2\Omega}$ ← Euler's formula

$\left(\cos\Omega = \frac{e^{j\Omega} + e^{-j\Omega}}{2} \right)$

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n}$$

$$= e^{j2\Omega} + e^{-j2\Omega}$$

$$x[n] = \begin{cases} 1 & n = 2 \\ 1 & n = -2 \\ 0 & \text{o.w.} \end{cases}$$

Warning: $\cos(1.5\Omega)$. $X(e^{j\Omega}) = ?$

using inspection

No

FT C.T., nonperiodic signals

$$\begin{cases} x(t) = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\,dt \\ X(j\omega) = \displaystyle\int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt \end{cases}$$

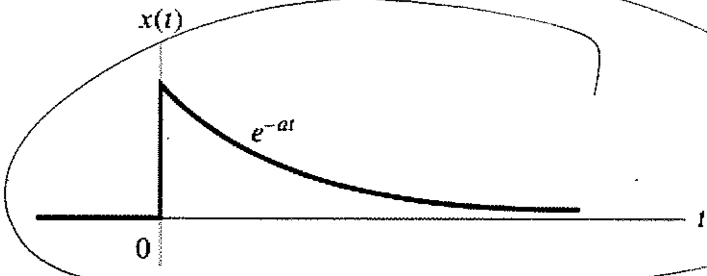
$$x(t) \xleftrightarrow{FT} X(j\omega)$$

DTFS, FS, DTFT, FT

E $x(t) = e^{-at}u(t)$. Find $X(j\omega)$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$$

Solution:



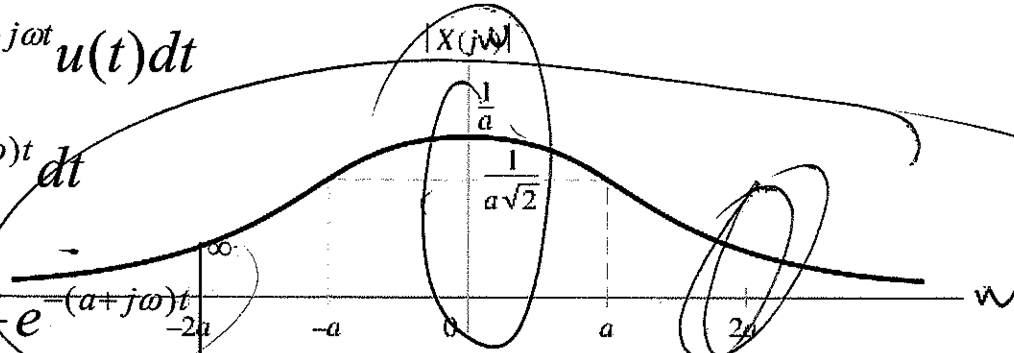
(a)

$$X(j\omega) = \int_{-\infty}^{\infty} e^{-at} e^{-j\omega t} u(t)\,dt$$

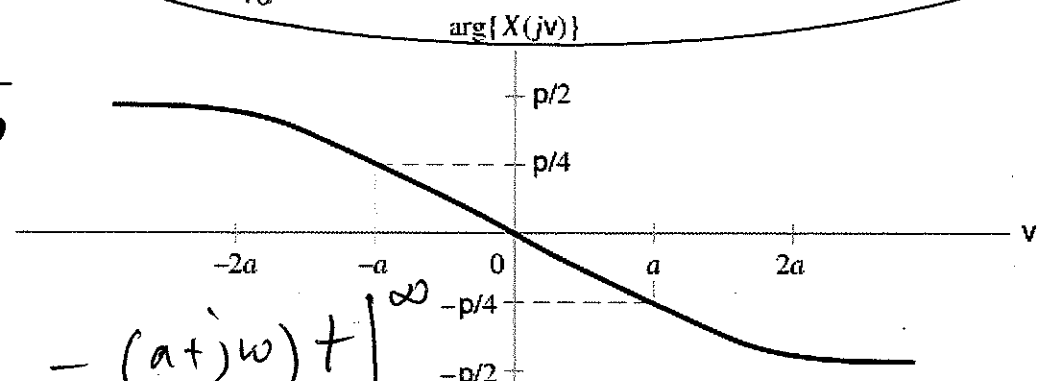
$$= \int_{0}^{\infty} e^{-(a+j\omega)t}\,dt$$

$$= -\frac{1}{a+j\omega} e^{-(a+j\omega)t} \Big|_{0}^{\infty}$$

$$= \frac{1}{a+j\omega}$$



(b)



(c)

$$-\frac{1}{a+jw} e^{-(a+jw)t} \Big|_{0}^{\infty}$$

E

Rectangular pulse: $x(t) = \begin{cases} 1, & -T_0 < t < T_0 \\ 0, & |t| > T_0 \end{cases}$ $\quad \operatorname{sinc} v = \frac{\sin \pi v}{\pi v}$

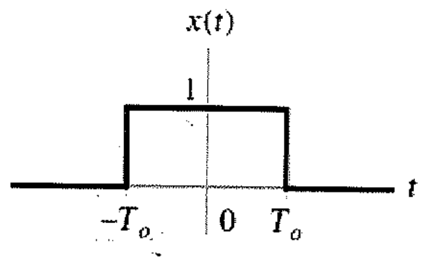
Solution: $X(j\omega) =$

$$\int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt = \int_{-T_0}^{T_0} e^{-j\omega t} dt$$

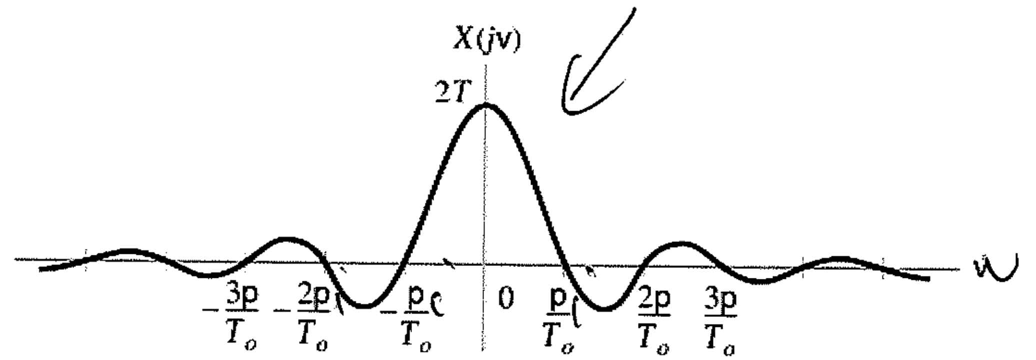
$$= -\frac{e^{-j\omega t}}{j\omega} \Big|_{-T_0}^{T_0} = \frac{e^{j\omega T_0} - e^{-j\omega T_0}}{j\omega}$$

$$= \frac{2 \sin \omega T_0}{\omega} = 2T_0 \frac{\sin \pi(\omega T_0/\pi)}{\pi\left(\frac{\omega T_0}{\pi}\right)}$$

$$= 2T_0 \operatorname{sinc}\left(\frac{\omega T_0}{\pi}\right)$$



(a)



(b)

$$2T_0 \operatorname{sinc}\left(\frac{\omega T_0}{\pi}\right)$$

$T_0 \uparrow \longrightarrow ?$

$T_0 \downarrow \longrightarrow ?$