Section 3.8

PROPERTIES OF FOURIER REPRESENTATIONS

Time property	Periodic (t,n)	Nonperiodic (t,n)	
C.T. (t)	• Fourier Series (FS) $x(t) = \sum_{k=-\infty}^{\infty} X[k]e^{jk\omega_0 t}, \omega_0 = \frac{2\pi}{T}$ (3.19) $X[k] = \frac{1}{T}\int_0^T x(t)e^{-jk\omega_0 t}dt,$ (3.20) (T: period)	• Four Transform (FT) $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega$ (3.35) $X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$ (3.36)	Non-periodic (k,ω)
D.T. [n]	• Discrete-Time Fourier Series (DTFS) $x[n] = \sum_{k=0}^{N-1} X[k]e^{jk\Omega_0 n}, \Omega_0 = \frac{2\pi}{N}$ (3.10) $X[k] = \frac{1}{N}\sum_{n=0}^{N-1} x[n]e^{-jk\Omega_0 n}$ (3.11) (N: period)	• Discrete-Time Fourier Transform (DTFT) $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\Omega})e^{j\Omega n}d\Omega$ (3.31) $X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$ (3.32)	Periodic (k,Ω)
	Discrete [k]	Continuous (ω, Ω)	Freq. property

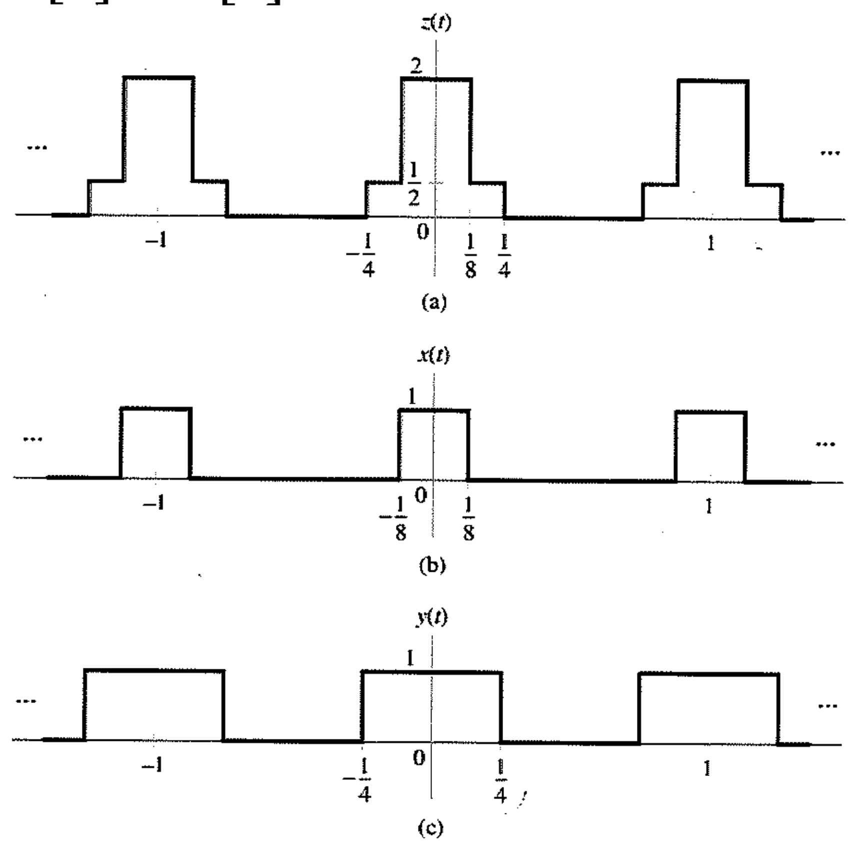
- <u>Linearity and symmetry</u>

$$z(t) = ax(t) + by(t) \xleftrightarrow{FT} Z(j\omega) = aX(j\omega) + bY(j\omega)$$
$$z(t) = ax(t) + by(t) \xleftrightarrow{FS;\omega_0} Z[k] = aX[k] + bY[k]$$
$$z[n] = ax[n] + by[n] \xleftrightarrow{DTFT} Z(e^{j\Omega}) = aX(e^{j\Omega}) + bY(e^{j\Omega})$$
$$z[n] = ax[n] + by[n] \xleftrightarrow{DTFS;\Omega_0} Z[k] = aX[k] + bY[k]$$

E Example 3.30, p255:

$$z(t) = \frac{3}{2}x(t) + \frac{1}{2}y(t)$$

Find the frequency-domain representation of z(t).



Which type of freq.-domain representation?
- FT, FS, DTFT, DTFS ?

Periodic signals, continuous time. Thus, FS.

$$Z[k] = \frac{3}{2} X[k] + \frac{1}{2} Y[k]$$

$$\begin{aligned}
X[k] &= \frac{1}{T_x} \int_0^{T_x} x(t) e^{-jk\omega_{0x}t}\, dt \\
&= \frac{1}{T_x} \int_{-T_x/2}^{T_x/2} x(t) e^{-jk\omega_{0x}t}\, dt \\
&= \int_{-1/8}^{1/8} e^{-jk2\pi t}\, dt = \frac{1}{-jk2\pi} e^{-jk2\pi t}\Big|_{-\frac{1}{8}}^{\frac{1}{8}} \\
&= \frac{1}{-jk2\pi}\left(-j2\sin\left(\tfrac{\pi}{4}k\right)\right) \\
&= \frac{1}{k\pi}\sin\left(\tfrac{\pi}{4}k\right)
\end{aligned}$$

$$Y[k] = \frac{1}{k\pi}\sin\left(\tfrac{\pi}{2}k\right)$$ ✓

$$\boxed{Z[k] \xleftrightarrow{FS;2\pi} \frac{3}{2k\pi}\sin\left(\tfrac{\pi}{4}k\right) + \frac{1}{2k\pi}\sin\left(\tfrac{\pi}{2}k\right)}$$

Symmetry:

We will develop using continuous, non-periodic signals. Results for other 2 cases may be obtained in a similar way.

a) Assume $x(t)$ real $\Rightarrow x^*(t) = x(t)$

$$X^*(j\omega) = \left[\int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt\right]^*$$

$$= \int_{-\infty}^{\infty} x^*(t)e^{j\omega t}dt$$

$$= \int_{-\infty}^{\infty} x(t)e^{-j(-\omega)t}dt$$

$$= X(-j\omega) = X(j(-\omega))$$

$X^*(j\omega) = X(j(-\omega))$

$$\boxed{\text{If } x(t) \text{ is real} \Rightarrow X(j\omega) \text{ is conjugate symmetric}}$$

√ Further assume $x(t)$ even (and real)

$$x(-t) = x(t),\ x^*(t) = x(t) \Rightarrow x^*(t) = x(-t)$$

$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt \Rightarrow X^*(j\omega) = \int_{-\infty}^{\infty} x^*(t)e^{j\omega t}dt$

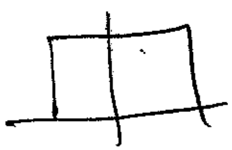
$$X^*(j\omega) = \int_{-\infty}^{\infty} x(-t)e^{-j\omega(-t)}dt$$

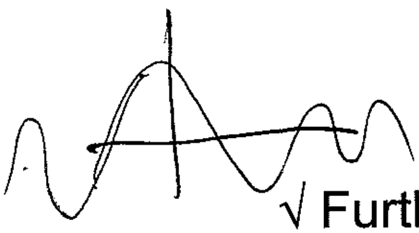
replace τ with $-t$ $\quad = \int_{-\infty}^{\infty} x(\tau)e^{-j\omega\tau}d\tau$ (why?)

$$= X(j\omega)$$

$$\boxed{\begin{aligned}&\int_a^b x(t)dt = -\int_b^a x(t)dt \\ &\tau = -t \Rightarrow d\tau = -dt\end{aligned}}$$

$x(t)$





$\boxed{\text{If } x(t) \text{ is real and even} \Rightarrow X(j\omega) \text{ is real.}}$

√ Further assume $x(t)$ odd (and real)

$$x(-t) = -x(t), x^*(t) = x(t) \Rightarrow x^*(t) = -x(-t)$$

$$X^*(j\omega) = \int_{-\infty}^{\infty}(-x(-t))e^{-j\omega(-t)}dt$$

$$= -X(j\omega)$$

$\boxed{\text{If } x(t) \text{ is real and odd} \Rightarrow X(j\omega) \text{ is purely imaginary.}}$