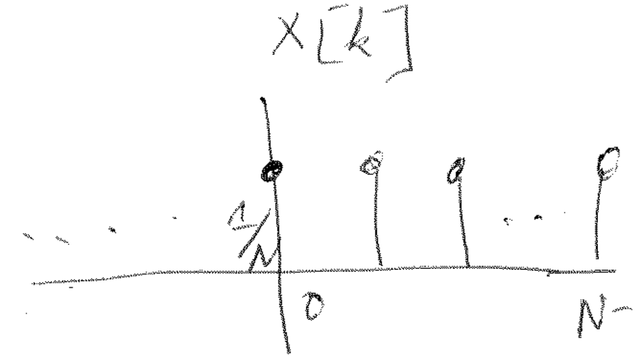
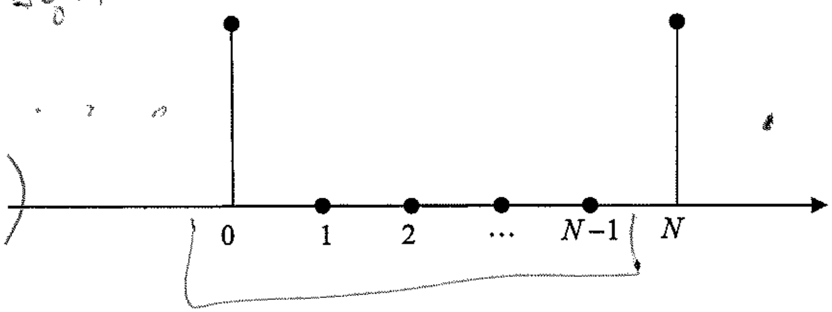


$\boxed{\text{E}}$

DTFS of an Impulse train: $x[n] = \sum_{l=-\infty}^{\infty} \delta[n - lN]$

Solution: $X[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-jk\Omega_0 n}$

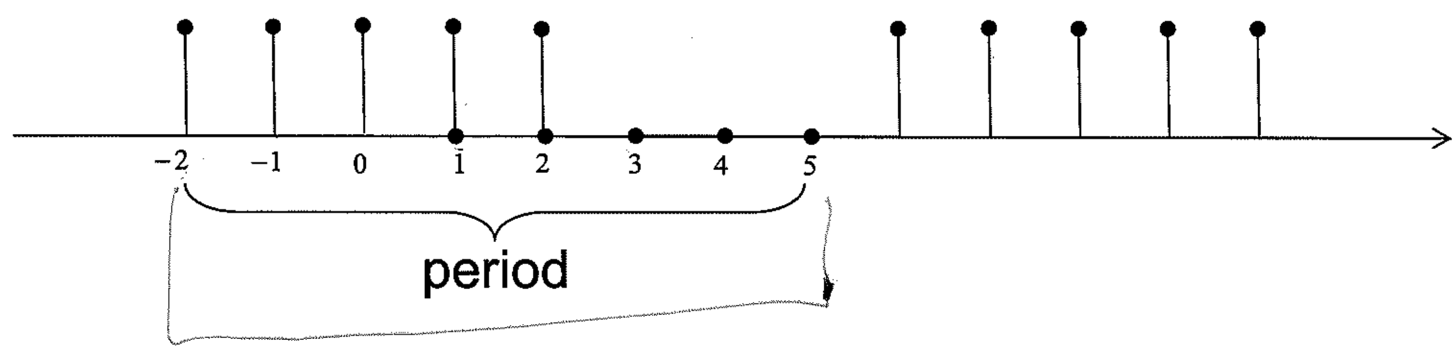
$= \frac{1}{N} \quad (k = 0, 1, 2, \ldots N-1)$



$\boxed{\text{E}}$ DTFS of a square signal: train

$$x[n] = \begin{cases} 1, & -M \le n \le M \\ 0, & M < n < N - M \end{cases}$$

Example $\begin{cases} M = 2 \\ N = 8 \end{cases}$



x[n]

period: $N = 8$

Solution:

$$X[k] = \frac{1}{N} \sum_{n=-2}^{5} x[n] e^{-jk\frac{2\pi}{N}n}$$

$$= \frac{1}{8}\left[(1)e^{-j\frac{k\pi}{4}\cdot(-2)} + (1)\left(e^{-j\frac{k\pi}{4}(-1)}\right) + (1)e^{-j\frac{k\pi}{4}(0)} + (1)\left(e^{-j\frac{k\pi}{4}(1)}\right) + (1)e^{-j\frac{k\pi}{4}(2)}\right]$$

$$= \frac{1}{8} e^{j\left(\frac{k\pi}{4}\right)(2)}\left[1 + e^{-j\frac{k\pi}{4}} + e^{-j\frac{k\pi}{4}(2)} + e^{-j\frac{k\pi}{4}(3)} + e^{-j\frac{k\pi}{4}(4)}\right]$$

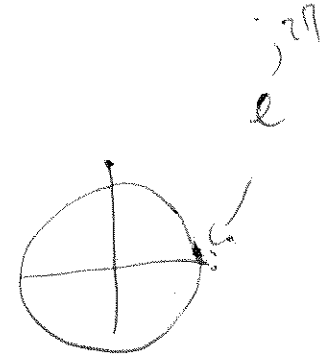
$$= \frac{1}{8} e^{j\frac{k\pi}{4}(2)}\left[\frac{1 - e^{-j\frac{k\pi}{4}5}}{1 - e^{-j\frac{k\pi}{4}}}\right]$$

Inverse of DTFS.

E One period of DTFS coefficients

$$X[k] = \left(\frac{1}{2}\right)^k, \quad 0 \le k \le 9$$

Determine $x[n]$ assuming $N = 10$



Solution: Use definition method.

$$x[n] = \sum_{k=0}^{N-1} X[k] e^{jk\Omega_0 n}$$

$$= \sum_{k=0}^{9} \left(\frac{1}{2}\right)^k e^{jk\frac{2\pi}{10}n} = \sum_{k=0}^{9} \left(\frac{1}{2} e^{j\frac{2\pi n}{10}}\right)^k$$

$$= \frac{1 - \left(\frac{1}{2} e^{j\frac{2\pi n}{10}}\right)^{10}}{1 - \frac{1}{2} e^{j\frac{\pi}{5}n}} = \frac{1 - \left(\frac{1}{2}\right)^{10} e^{j2\pi n}}{1 - \left(\frac{1}{2}\right) e^{j\frac{\pi}{5}n}} = \frac{1 - \left(\frac{1}{2}\right)^{10}}{1 - \left(\frac{1}{2}\right) e^{j\frac{\pi}{5}n}}$$

$n = 0, \cdots, 9$

$x[13] = x[3]$

$x[13] = x[13-10] = x[3]$

F.S. (CT, periodic). x(t): fundamental period T
fundamental frequency

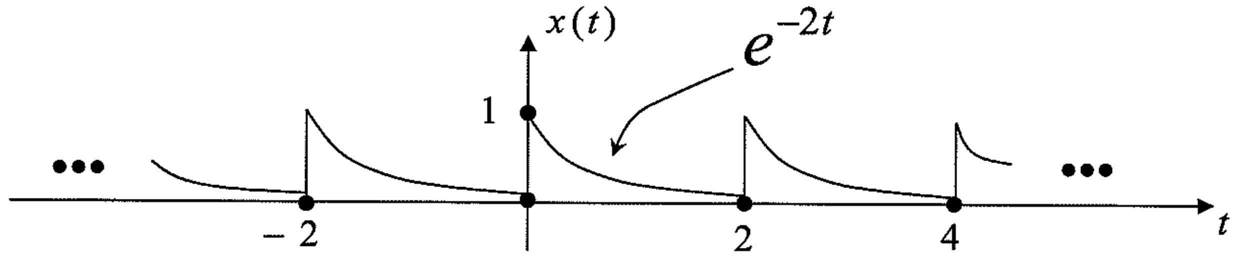
$$\omega_0 = 2\pi/T$$

$$\begin{cases} x(t) = \sum_{k=-\infty}^{\infty} X[k] e^{jk\omega_0 t} & (*) \\ X[k] = \frac{1}{T}\int_0^T x(t) e^{-jk\omega_0 t}\,dt \end{cases}$$

x(t) and X[k] are an FS pair: $\boxed{x(t) \xleftrightarrow{FS;\omega_0} X[k]}$

FS coefficients X[k] are a freq-domain representation of x(t).

$\boxed{E}$ x(t) given as



Solution: $X[k] = \frac{1}{T}\int_0^T x(t)\, e^{-jk\omega_0 t}\, dt$ $\quad T = 2$, $\omega_0 = \frac{2\pi}{T} = \pi$

$$X[k] = \frac{1}{2}\int_0^2 e^{-2t}\, e^{0-jk\pi t}\, dt = \frac{1}{2}\int_0^2 e^{-t(2+jk\pi)}\, dt$$

$$= \frac{1}{2}\left(\frac{e^{-t(2+jk\pi)}}{-(2+jk\pi)}\right)\Bigg|_0^2 = \frac{1}{2}\left(\frac{e^{-2(2+jk\pi)}}{-(2+jk\pi)} + \frac{1}{2+jk\pi}\right)$$

$$= \frac{1 - e^{-4}e^{-2jk\pi}}{4 + 2\pi jk} = \frac{1 - e^{-4}}{4 + 2\pi jk}$$

$$X[k] = \frac{1}{T}\int_0^T x(t)e^{-jk\omega_0 t}dt \qquad \text{where} \begin{cases} T = 2 \\ \omega_0 = \pi = \dfrac{2\pi}{T} \end{cases}$$

$$= \frac{1}{2}\int_0^2 e^{-2t}e^{-j\pi kt}dt$$

$$= \frac{1}{2}\int_0^2 e^{-(2+j\pi k)t}dt$$

$$|a+jb| = \sqrt{a^2+b^2}$$

$$= \frac{-1}{2(2+j\pi k)}e^{-(2+j\pi k)t}\Big|_0^2$$

$$= \frac{1}{4+j2\pi k}\left(1-e^{-4}\underbrace{e^{-j2\pi k}}_{=1}\right)$$

$$= \frac{1-e^{-4}}{4+j2\pi k} = (1-e^{-4})\left[\frac{4-j2\pi k}{(4+j2\pi k)(4-j2\pi k)}\right]$$

$$|X[k]| = \frac{(1-e^{-4})(4-j2\pi k)}{16+4\pi^2k^2}$$

$$X[k]$$

$$|X[k]| = \frac{(1-e^{-4})}{\sqrt{16+4\pi^2k^2}}$$