E Determine X[k] of $x(t) = \sum_{l=-\infty}^{\infty} \delta(t-4l)$

Solution:

$T = 4 \qquad w_0 = \frac{2\pi}{T}$

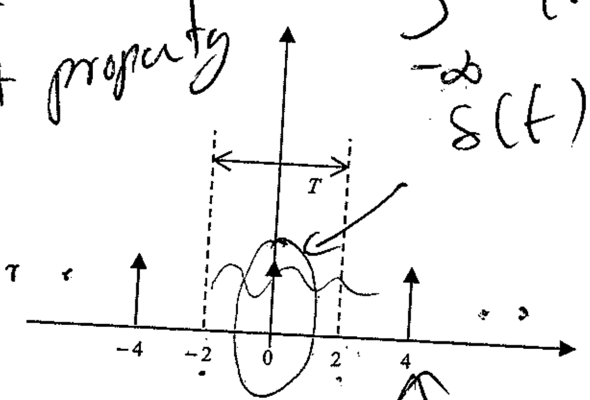
$$X[k] = \frac{1}{T}\int_{0}^{T} x(t) e^{-jkw_0 t}\, dt = \frac{1}{T}\int_{-T/2}^{T/2} x(t) e^{-jkw_0 t}\, dt$$

$$= \frac{1}{4}\int_{-2}^{2} \delta(t) e^{-jk\frac{\pi}{2}t}\, dt$$

sift property

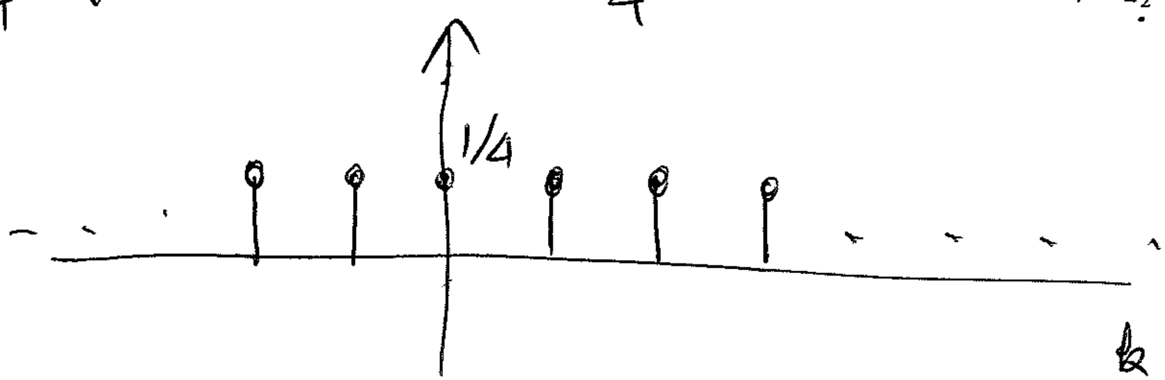
$$\int_{-\infty}^{\infty} \delta(t-t_0)\, x(t)\, dt = x(t_0)$$

$\delta(t)$



$\delta(t-4)$

$$= \frac{1}{4}\left(e^{-jk\frac{\pi}{2}(0)}\right) = \frac{1}{4}$$



FS coefficients by inspection.

$\boxed{\text{E}}$ $x(t) = 3\cos\left(\frac{\pi}{2}t + \frac{\pi}{4}\right)$ Find X[k]

$$x(t) = \sum_{k=-\infty}^{\infty} X[k] e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} X[k] e^{jk(\pi/2)t}$$

Solution:

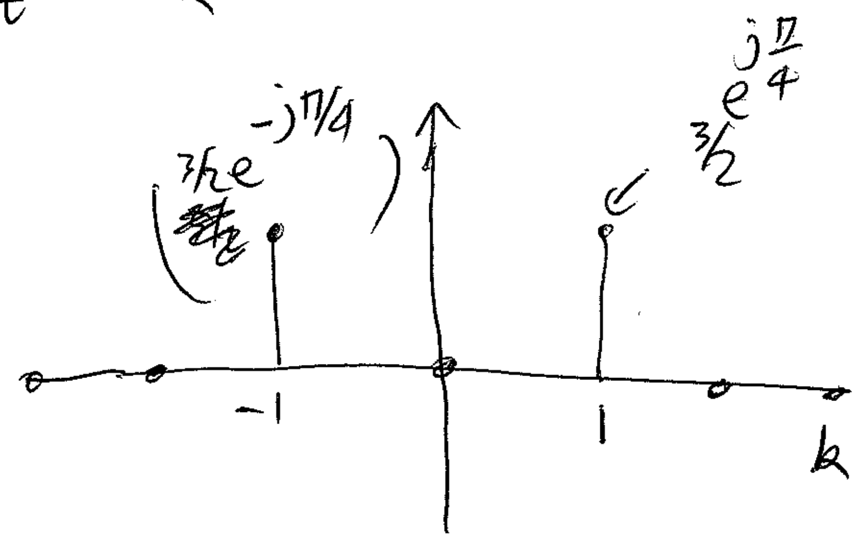
$$3\cos\left(\frac{\pi}{2}t+\frac{\pi}{4}\right) = 3\left(\frac{e^{j\left(\frac{\pi}{2}t+\frac{\pi}{4}\right)} + e^{-j\left(\frac{\pi}{2}t+\frac{\pi}{4}\right)}}{2}\right)$$

$$= \left(\frac{3}{2}e^{j\frac{\pi}{4}}\right)e^{j\frac{\pi}{2}t} + \left(\frac{3}{2}e^{-j\frac{\pi}{4}}\right)e^{-j\frac{\pi}{2}t}$$

$$\omega_0 = \frac{\pi}{2}$$

$$= \sum_{k=-\infty}^{\infty} X[k]\, e^{jk\frac{\pi}{2}t}$$

$$X[k] = \begin{cases} \frac{3}{2}e^{j\frac{\pi}{4}}, & k = 1 \\ \frac{3}{2}e^{-j\frac{\pi}{4}}, & k = -1 \\ 0, & \text{o.w.} \end{cases}$$



E $x(t)=2\sin(2\pi t-3)+\cancel{4}\sin(6\pi t)$. Find $X[k]$

$$\boxed{x(t)=\sum_{k=-\infty}^{\infty} X[k]e^{jk\omega_0 t}}$$

<u>Solution:</u> $\omega_0 = 2\pi$

$X[k]$: ?

$$2\sin(2\pi t-3) + \sin(6\pi t) = 2\left[\frac{e^{+j(2\pi t-3)} - e^{-j(2\pi t-3)}}{2j}\right] + \frac{e^{j6\pi t} - e^{-j6\pi t}}{2j} = \left(\frac{1}{j}e^{-3j}\right)e^{j2\pi t} + \left(-\frac{1}{j}e^{3j}\right)e^{-j2\pi t} + \left(\frac{1}{2j}\right)e^{j6\pi t} - \left(\frac{1}{2j}\right)e^{-j6\pi t}$$

$$= \sum_{k=-\infty}^{\infty} X[k]e^{jk2\pi t}$$

$$X[k] = \begin{cases} 0 & \text{ow} \\ \frac{1}{j}e^{-3j}, & k=1 \\ -\frac{1}{j}e^{3j}, & k=-1 \\ \frac{1}{2j}, & k=3 \\ -\frac{1}{2j}, & k=-3 \end{cases}$$

E Inverse FS.

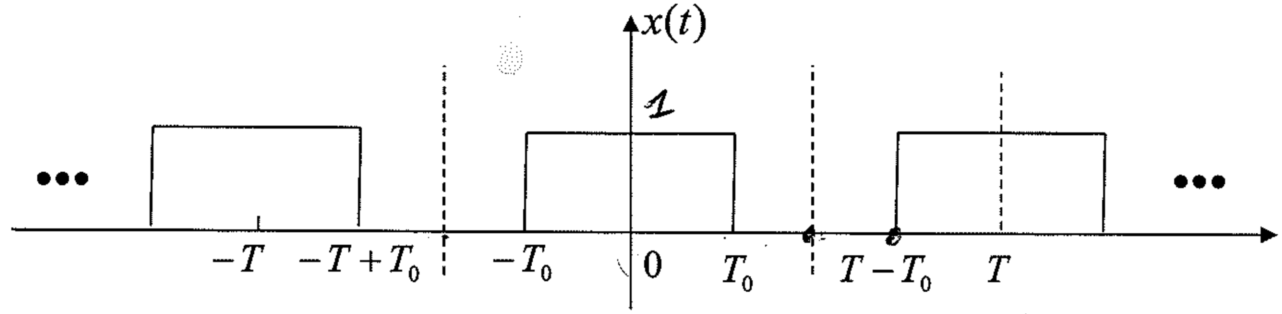
$$X[k] = -j\delta[k-2] + j\delta[k+2] + 2\delta[k-3] + 2\delta[k+3],\ \omega_0 = \pi.\ \text{Find } x(t)$$

<u>Solution:</u>

$$x(t) = \sum_{k=-\infty}^{\infty} X[k] e^{jk\omega_0 t}$$
$$= \left(-je^{j2\pi t} + je^{-j2\pi t}\right) + \left(2e^{j3\pi t} + 2e^{-j3\pi t}\right)$$
$$= 2\sin(2\pi t) + 4\cos(3\pi t)$$

$$-je^{j2\pi t} + je^{-j2\pi t} = j2\left(\frac{-je^{j2\pi t} + je^{-j2\pi t}}{2j}\right)$$
$$= \frac{2\left(e^{j2\pi t} - e^{-j2\pi t}\right)}{2j} = 2\sin 2\pi t$$

$\boxed{E}$ FS of a square wave.



Period is T, so $\omega_0 = 2\pi/T$

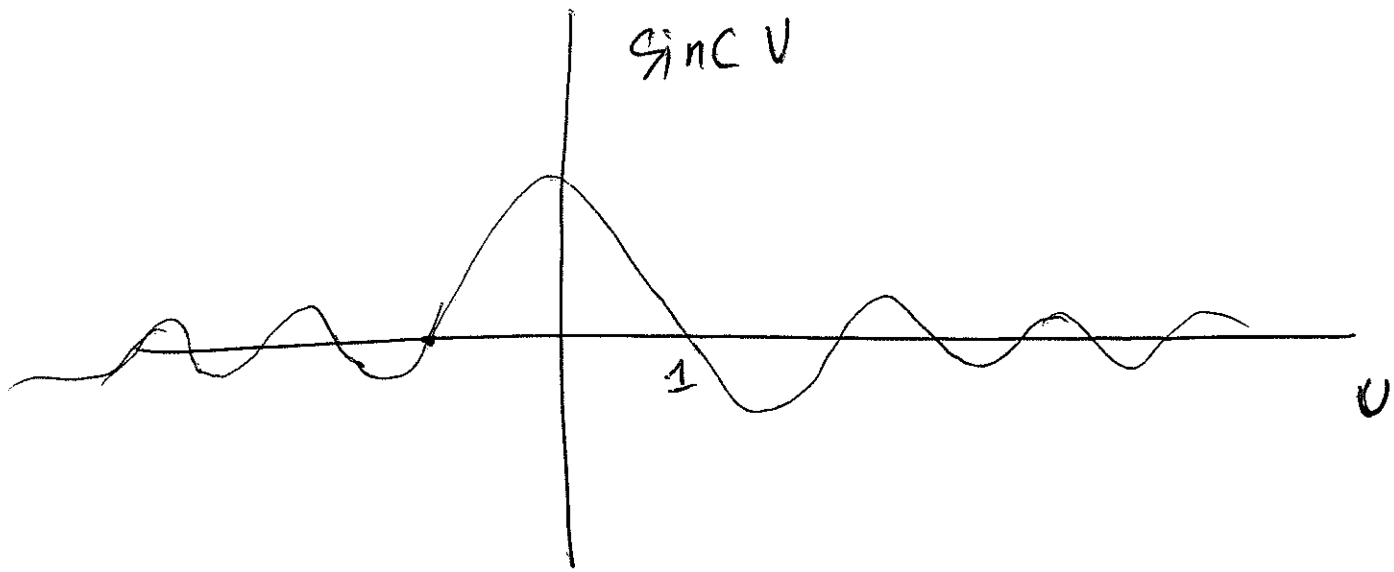
Solution: $$X[k] = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-jk\omega_0 t}\, dt = \frac{1}{T}\int_{-T_0}^{T_0} e^{-jk\frac{2\pi}{T}t}\, dt$$

$$= \frac{1}{T}\left(\frac{e^{-jk\frac{2\pi}{T}t}}{-\left(jk\frac{2\pi}{T}\right)}\right)\Bigg|_{-T_0}^{T_0} = -\frac{1}{T}\left(\frac{e^{-jk\frac{2\pi T_0}{T}} - e^{jk\frac{2\pi T_0}{T}}}{jk\frac{2\pi}{T}}\right)$$

$$= \frac{2}{T}\,\frac{\sin\left(k\frac{2\pi T_0}{T}\right)}{k\left(\frac{2\pi}{T}\right)} = \frac{2T_0}{T}\,\mathrm{sinc}\left(k\frac{2T_0}{T}\right)$$

$$\mathrm{sinc}\, u \triangleq \frac{\sin \pi u}{\pi u}$$

$$u = \frac{k2T_0}{T}$$


sinc v
1
v