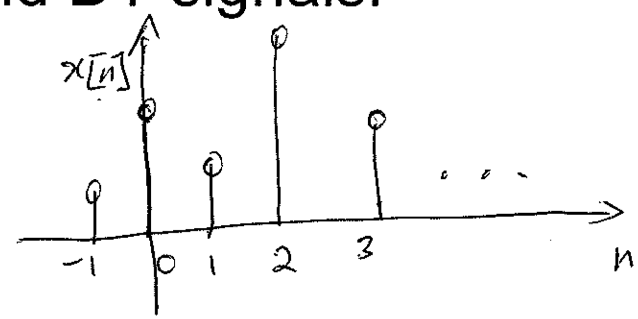
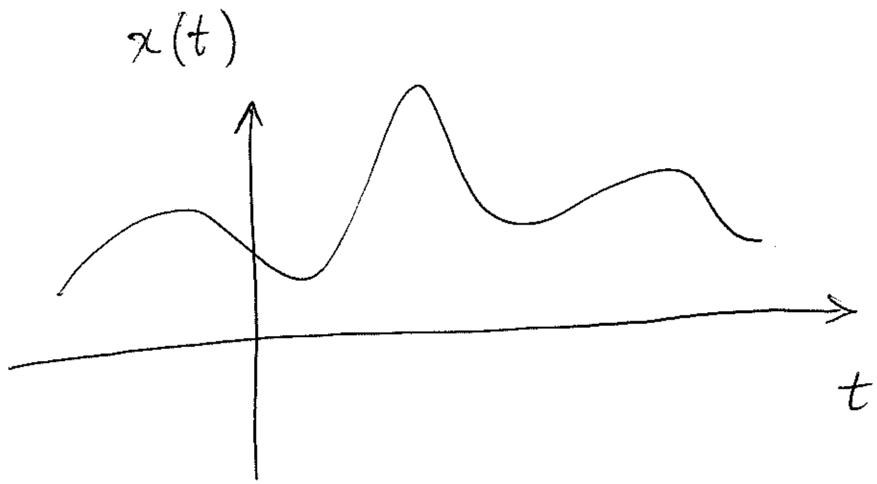
Classification of signals

- Based on features:

1. CT and DT signals:





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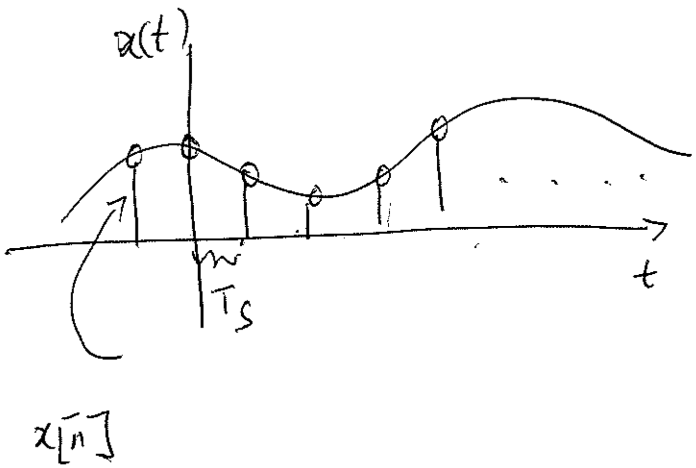
Classification of signals (cont.)

- For many cases, $x[n]$ is obtained by sampling $x(t)$ as:
  $x[n] = x(nT_s) = x(t)|_{t=nT_s}, \ n = -\infty, \cdots, 0, \cdots, \infty$
- $x(t)$ must be recoverable from $x[n]$
- Are there any requirements for the sampling?

$x(t)$ can be recovered from $x[n]$ if the sampling frequency is just enough (Nyquist frequency)



Classification of signals (cont.)

2. Even and odd signals:

<u>Even</u>:
$$\begin{cases} x(t) = x(-t) & \text{CT} \\ x[n] = x[-n] & \text{DT} \end{cases}$$

<u>odd</u>:
$$\begin{cases} x(t) = -x(-t) & \text{CT} \\ x[n] = -x[-n] & \text{DT} \end{cases}$$

- An arbitrary signal $x(t)$ can be expressed as

$$x(t) = \underbrace{x_e(t)}_{\text{even signal}} + \underbrace{x_o(t)}_{\text{odd signal}}$$

$$x_e(t) = \frac{x(t) + x(-t)}{2} \quad ; \quad x_o(t) = \frac{x(t) - x(-t)}{2}$$

note that: $x_e(t) = x_e(-t)$; $x_o(t) = x_o(-t)$

$$* \quad x_e(t) + x_o(t) = \frac{x(t) + x(-t)}{2} + \frac{x(t) - x(-t)}{2} = x(t)$$

Classification of signals (cont.)

- $x(t)$ is conjugate symmetric if $x(-t) = x^*(t)$, where $x(t) = a(t) + jb(t)$, $j = \sqrt{-1}$ and $a(t)$ and $b(t)$ are real.

$$\begin{cases} x^*(t) = a(t) - jb(t) \\ x(-t) = a(-t) + jb(-t) \end{cases}$$

- If $\Re\{x(t)\}$ is even and $\Im\{x(t)\}$ is odd, then $x(t)$ is conjugate symmetric. prove it yourself.

Classification of signals (cont.)

3. Periodic and non-periodic signals:
   CT signal: if $x(t) = x(t + T_p),\ \forall t$, then $x(t)$ is periodic.

   - Fundamental period: T_p
   - Fundamental frequency $f_p = 1/T_p$ (Hz or cycles/second)
   - Angular frequency: $\omega_p = 2\pi f_p = 2\pi/T_p$ (rad/seconds)

   DT signal: if $x[n] = x[n + N_p],\ \forall n$, then $x[n]$ is periodic.

   - N_p: $N_p > 0$ integer. $\min(N_p)$: fundamental period
   - N_p: samples/period, if the unit of n is designated as samples.
   - $F_p = 1/N_p$ (cycles/sample)
   - $\Omega_p = 2\pi F_p$ (rads/sample). If the unit of n is designated as dimensionless, then Ω_p is simply in radians.

   Note: A sampled CT periodic signal may not be DT periodic.
   Condition for DT signals to be periodic:

   - $x[n] = x(nT_s)$, Assume that after sampling, we have a periodic DT signal with period N_p, then

$$x[n] = x[n+N_p] = x((n+N_p)T_s) = x(nT_s + N_pT_s) = x(nT_s)$$

$$\Rightarrow N_p T_s = k T_p \Rightarrow \boxed{T_s = k\frac{T_p}{N_p}} \Rightarrow \boxed{\frac{T_p}{T_s} = \frac{N_p}{k}}$$

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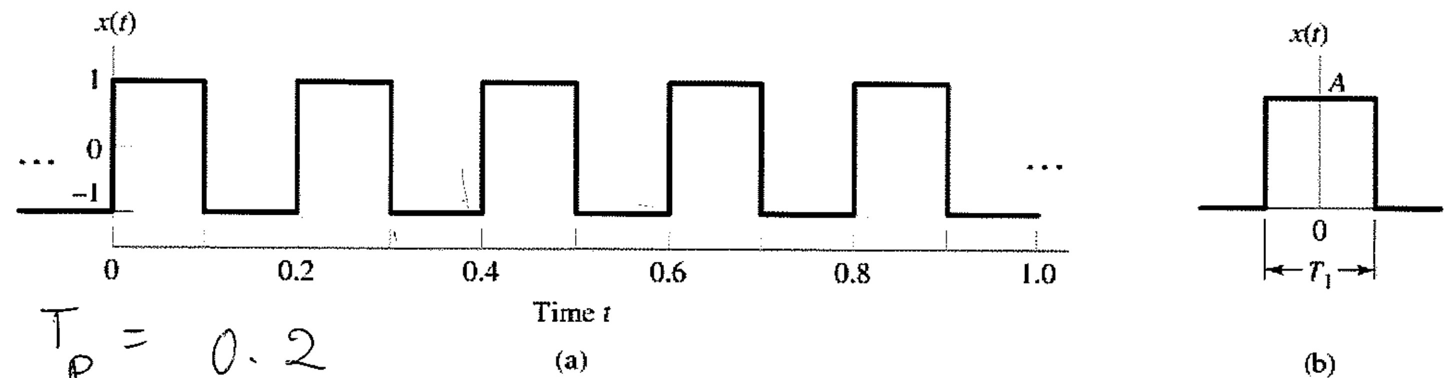
Classification of signals (cont.)

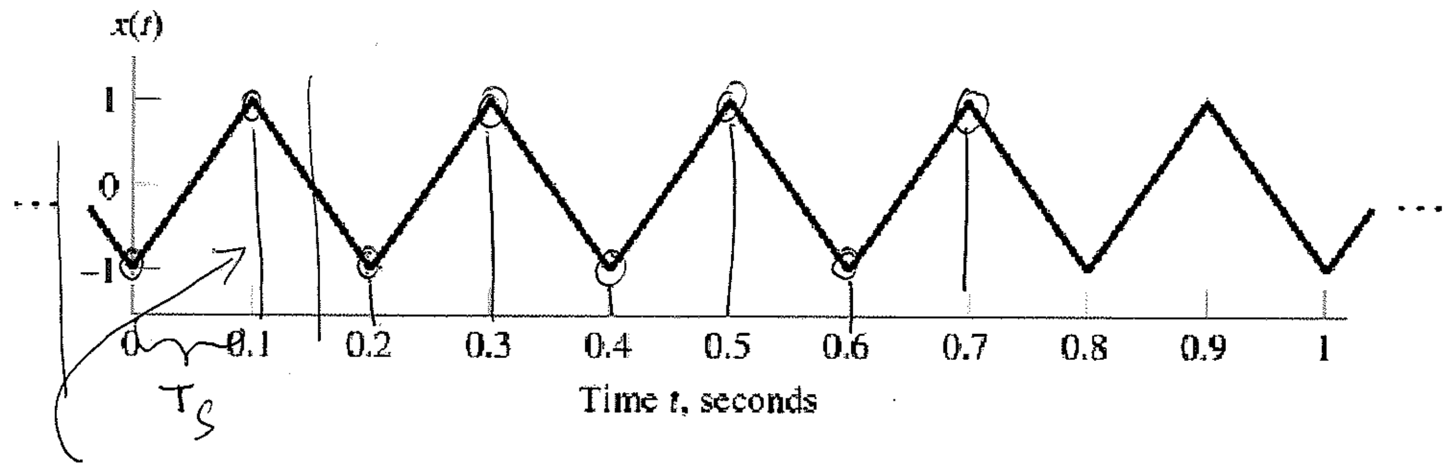


$T_p = 0.2$

$f_p = \frac{1}{0.2} = 5$

(a)

$\omega_p = 2\pi f_p = 10\pi$



$x[n]$

$N_p = 2$

(b)

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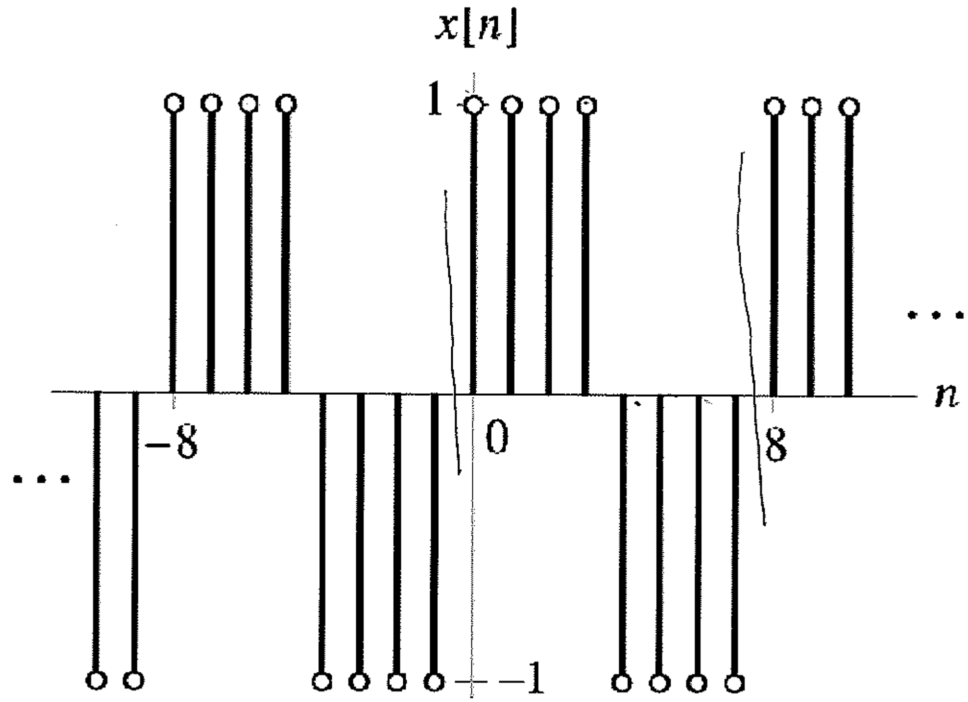
Classification of signals (cont.)

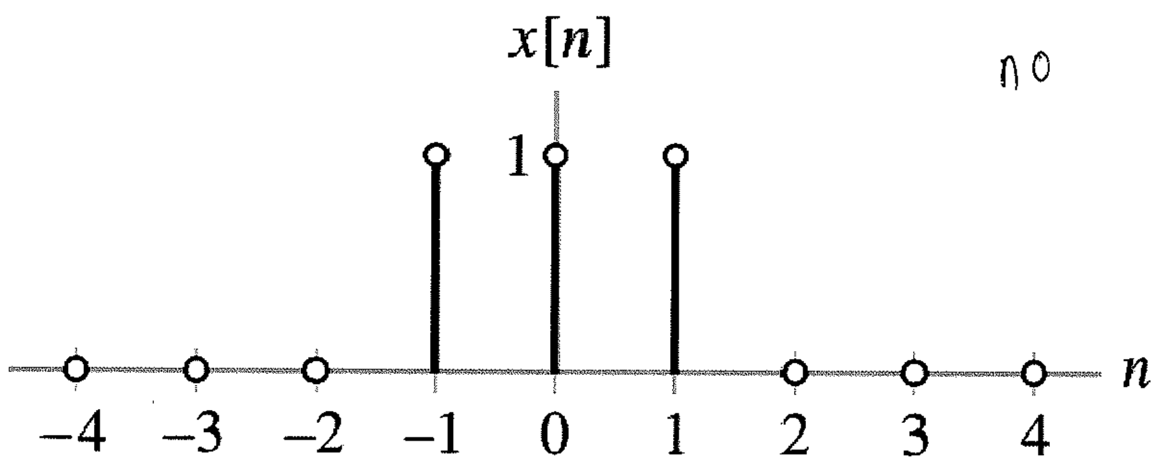


$N_p = 8$

$F_p = \frac{1}{N_p} = \frac{1}{8}$

$\Omega_p = 2\pi F_p = \frac{2\pi}{8} = \frac{\pi}{4}$

(c)



n0

(d)

Classification of signals (cont.)

E: $x[n] = A\cos(2\pi F_p n + \theta)$. Condition for $x[n]$ to be periodic?

Suppose $x[n] = x[n+N_p] = A\cos(2\pi F_p(n+N_p)+\theta) = A\cos(2\pi F_p n + 2\pi F_p N_p + \theta)$

$= A\cos(2\pi F_p n + \theta)$

$\Rightarrow 2\pi F_p N_p = 2\pi k$

$\Rightarrow F_p = \frac{k}{N_p} \Rightarrow F_p$ has to be a rational number.

E: $x(t) = \sin^2(20\pi t)$ periodic? non-periodic?