Classification of signals (cont.)

E: $x[n] = A\cos(2\pi F_p n + \theta)$. Condition for $x[n]$ to be periodic?

$$F_p = \frac{k}{N_p}$$

E: $x(t) = \sin^2(20\pi t)$ periodic? non-periodic?

$$\boxed{\sin^2\theta = \frac{1 - \cos 2\theta}{2}}$$

$$x(t) = \frac{1}{2} - \frac{\cos 40\pi t}{2}$$

so $x(t)$ is periodic

$$\omega_p = 40\pi$$

$$f_p = \frac{\omega_p}{2\pi} = \frac{40\pi}{2\pi} = 20 \text{ Hz}$$

$$T_p = \frac{1}{f_p} = \frac{1}{20} \text{ s}$$

Classification of signals (cont.)

- If $x(t)$ is sampled at $t = nT_s$ with $T_s = \frac{1}{4\pi} \longrightarrow$

$T_p = \frac{1}{20}$, $T_s = \frac{1}{4\pi}$,

$\frac{T_p}{T_s} = \left(\frac{1}{20}\right)(4\pi) = \frac{\pi}{5} \neq$ not rational number , so

$x[n]$ will not be periodic

- If $x(t)$ is sampled at $t = nT_s$ with $T_s = \frac{1}{40} \longrightarrow$

$\frac{T_p}{T_s} = \left(\frac{1}{20}\right)(40) = 2 = \text{rational}$ so

$x[n]$ will be periodic

Classification of signals (cont.)

Sum of signals

CT signal:

If $x_1(t)$ and $x_2(t)$ are periodic with fundamental periods T_1 and T_2. How about $x(t) = x_1(t) + x_2(t)$?

$x_1(t) = x_1(t+T_1)$, $x_2(t) = x_2(t+T_2)$,

Suppose T_{sum} is the period of $x(t)$, then

$x(t+T_{sum}) = x_1(t+T_{sum}) + x_2(t+T_{sum}) = x(t) = x_1(t) + x_2(t)$

$\Rightarrow \left.\begin{array}{l} T_{sum} = kT_1 \\ T_{sum} = lT_2 \end{array}\right\} \Rightarrow$ T_{sum} is choosen using smallest (k, l) such that $kT_1 = lT_2$. (Note: k, l are positive integers)

E: $x_1(t) = \cos(\pi t/2)$, $x_2(t) = \cos(\pi t/3)$, $x_1(t) + x_2(t)$

$\omega_1 = \frac{\pi}{2}$

$f_1 = \frac{\omega_1}{2\pi} = \frac{\pi}{2}\frac{1}{2\pi} = \frac{1}{4}$

$T_1 = \frac{1}{f_1} = 4$

$\omega_2 = \pi/3$

$f_2 = \frac{\omega_2}{2\pi} = \frac{\pi}{3}\left(\frac{1}{2\pi}\right) = \frac{1}{6}$

$T_2 = 1/f_2 = 6$

$\left.\begin{array}{l} kT_1 = lT_2 \\ k4 = l6 \end{array}\right\} \Rightarrow \begin{array}{l} k = 3 \\ l = 2 \end{array}$, $T_{sum} = (3)(4) = 12$

Classification of signals (cont.)

DT signal:

$$\text{If} \begin{cases} x_1[n] = x_1[n+N_1] \\ x_2[n] = x_2[n+N_2] \end{cases}$$

$$x_1[n] + x_2[n] = x_1[n+N_{sum}] + x_2[n+N_{sum}], \forall n$$

Smallest k and l such that

$$kN_1 = lN_2$$

$$N_{sum} = kN_1 = lN_2$$

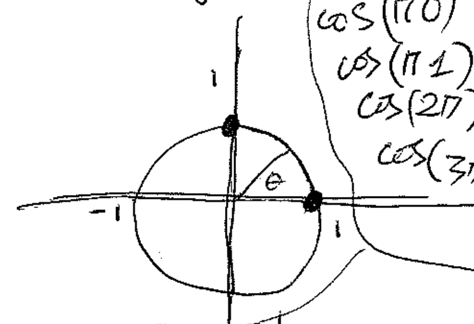
ex: $x_1[n] = \cos(\pi n)$, $x_2[n] = \sin\left(\frac{\pi}{2} n\right)$, Find the period

N_{sum} of $x[n] = x_1[n] + x_2[n]$

$$\left.\begin{matrix} N_1 = 2 \\ N_2 = 4 \end{matrix}\right\} \Rightarrow k2 = l4$$

$$\Rightarrow k = 2, \; l = 1$$

$$N_{sum} = 4$$



$\cos(\pi 0) = 1$
$\cos(\pi 1) = -1$
$\cos(2\pi) = 1$
$\cos(3\pi) = -1$

$\sin(0) = 0$
$\sin\left(\frac{\pi}{2}(1)\right) = 1$
$\sin\left(\frac{\pi}{2} 2\right) = 0$

$\sin\left(\frac{\pi}{2} 3\right) = -1$
$\sin(2\pi) = 0$

Classification of signals (cont.)

5. Energy and power signals:

- CT signal $x(t)$:

$$\text{Energy: } E = \int_{-\infty}^{\infty} x^2(t)dt$$

$$\text{Power: } P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)dt \quad - \quad \text{non-periodic signal}$$

$$P = \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)dt \quad - \quad \text{periodic signal}$$

T : period.

Classification of signals (cont.)

- DT signal $x[n]$:

Energy: $E = \sum_{n=-\infty}^{\infty} x^2[n]$

Power: $P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x^2[n]$ – non-periodic signa

$P = \frac{1}{N} \sum_{n=0}^{N-1} x^2[n]$ – periodic signal

- Energy signal: iff $0 < E < \infty$
- Power signal: iff $0 < P < \infty$

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__example:__ $x[n] = \frac{1}{|n|+1}$, $n \neq 0$ E ? P ?

$x^2[n] = \frac{1}{(|n|+1)^2}$

$E = \sum_{n=-\infty}^{\infty} \frac{1}{(|n|+1)^2} < \infty$ $\sum_{n=-\infty}^{\infty} \frac{1}{|n|^p}$

$p \geq 1 \rightarrow$ converge

$p < 1 \rightarrow$ ~~converge~~ diverges

$x[n]$ is energy signal.