ECE351: Signals and Systems - Dr. Thinh Nguyen
Midterm 2
Feb. 27, 2015

Instruction: Please write your work clearly. Credits will not be given to the correct answers without proper derivations. You are allowed a 2-sided 8.5×11" sheet of notes. No calculator is allowed. Answers should not contain the symbols for integration or sum. DO NOT use any Fourier representation pairs in the book appendix for your calculations. You have 50 minutes to do the exam.

1. **Problem 1 (20pts)**

   Let $x(t)$ be a periodic continuous time signal with the fundamental frequency of $4,400$ Hz and $X[k] = \frac{k}{1+jk}$.

   (a) Determine the energy level of the $x(t)$ (intuitively represented by $|X[k]|$) at frequency 4,400 Hz (15pts).

   (b) Determine the energy level of the signal $x(t)$ at frequency 8900 Hz (5 pts).

2. **Problem 2 (30pts)**

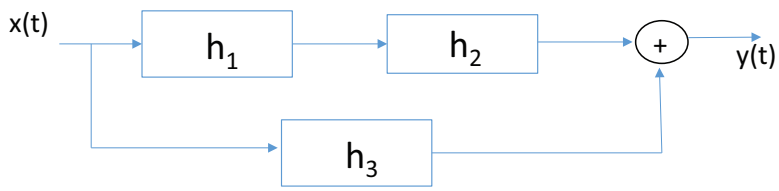

Figure 1: Problem 2

An LTI system consisting of three LTI subsystems are shown in the figure above. Let the frequency responses of the three subsystems be defined as:

$$H_1(j\omega) = \begin{cases} |\omega| & |\omega| < \pi \\ 0 & \text{otherwise.} \end{cases}, \quad H_2(j\omega) = \begin{cases} 1 & \frac{\pi}{2} < |\omega| < 2\pi \\ 0 & \text{otherwise.} \end{cases}, \quad H_3(j\omega) = \begin{cases} -1 & \pi < |\omega| < 2\pi \\ 0 & \text{otherwise.} \end{cases}$$

Let $x(t) = 2\sin\frac{3\pi}{4}t + 3\cos\frac{3\pi}{2}t$ be the input into the system, determine the output $y(t)$.

3. **Problem 3 (30pts)**

   Let $X[k] = 2\cos\frac{3\pi}{5}k + \sin\frac{3\pi}{2}k$.

   (a) Determine the fundamental period of $X[k]$ (10pts)

   (b) Determine the corresponding time domain signal $x[n]$. (20pts)

4. **Problem 4 (20pts)**

   Let

$$z(t) = \begin{cases} \sum_{i=0}^{i=K} a^i \delta(t-i) & 0 \le t \le K \\ 0 & \text{otherwise} \end{cases},$$

   where K and a are some positive constants. Let $x(t) = \sum_{m=-\infty}^{\infty} z(t-(K+1)m)$.

   (a) Determine the period of $x(t)$ (5pts).

   (b) Determine and calculate the appropriate Fourier representation of $x(t)$ (15pts).