ECE351: Signals and Systems I - Sample 2 - Dr. Thinh Nguyen

Instruction: Please write your work clearly. Credits will not be given to the correct answers without proper derivations. You are allowed a 2-sided 8.5x11" sheet of notes. No calculator is allowed. Answers should not contain the symbols for integration or sum. You have 50 minutes to do the exam.

1. (30 pts) Determine the appropriate Fourier representation of

$$x[n] = (\frac{1}{2})^n \cos(\pi n) u[n]$$

2. (35 pts) Determine the appropriate Fourier representation of

$$x(t) = \sum_{n=-\infty}^{\infty} y(t-2n), \text{ where } y(t) = \begin{cases} t, & 0 \leq t \leq 1 \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

3. (35 pts) Let $x(t) = \cos(\pi t)$ be an input to an LTI system $h(t)$ with the frequency response $H(j\omega) = \frac{j\omega+1}{\omega^2+1}$. Determine the output $y(t)$ from the system. Make sure you simplify $y(t)$ as much as possible.

4. (10 pts) **Bonus question**. Circle ***all*** the true statements about the Fourier representation of $\frac{\sin(t)}{t^3}$.

   (a) It is purely real.

   (b) It is purely imaginary.

   (c) It has both real and imaginary components.

   (d) It is even.

① . $x[n] = \left(\frac{1}{2}\right)^n \cos(\pi n)\, u[n]$: DT non-periodic

use DTFT

$$X[e^{j\Omega}] = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n} = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n \left(\frac{e^{j\pi n} + e^{-jn\pi}}{2}\right) e^{-j\Omega n}$$

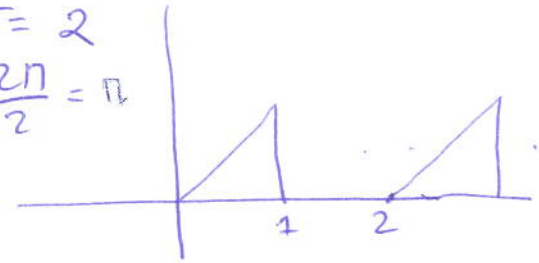
$$= \frac{1}{2} \sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right) e^{j(\pi - \Omega)}\right]^n + \frac{1}{2} \sum_{n=0}^{\infty} \left[\frac{1}{2} e^{-j(\pi+\Omega)}\right]^n$$

$$= \frac{1}{2} \frac{1}{\left(1 - \frac{1}{2} e^{j(\pi-\Omega)}\right)} + \frac{1}{2} \frac{1}{\left(1 - \frac{1}{2} e^{-j(\pi+\Omega)}\right)} = \frac{1}{1 + \frac{e^{-j\Omega}}{2}}$$

2) $x(t) = \sum_{n=-\infty}^{\infty} y(t-2n)$ where $y(t) = \begin{cases} t & 0 \le t \le 1 \\ 0, & \end{cases}$

CT periodic. use FS $T = 2$

$\omega_0 = \frac{2\pi}{2} = \pi$



$$X[k] = \frac{1}{2} \int_0^1 t e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{2} \int_0^1 t e^{-jk\pi t}\, dt = \frac{1}{2} \left[\left(\frac{e^{-jk\pi t}}{-jk\pi}\right) t - \int_0^1 \frac{e^{-jk\pi t}}{-jk\pi}\, dt\right]$$

$$= \frac{1}{2} \left[\left(\frac{e^{-jk\pi t}}{-jk\pi}\right) t + \frac{e^{-jk\pi t}}{k^2\pi^2}\right]\Bigg|_0^1$$

$$= \frac{1}{2} \left[\left(\frac{e^{-jk\pi}}{-jk\pi} + \frac{e^{-jk\pi}}{k^2\pi^2}\right) - \frac{1}{k^2\pi^2}\right]$$

3) $x(t) = \dfrac{e^{jnt} + e^{-jnt}}{2}$

$$y(t) = H(jn)\frac{e^{jnt}}{2} + \frac{H(-jn)\,e^{-jnt}}{2}$$

$$= \left(\frac{jn+1}{n^2+1}\right)\frac{e^{jnt}}{2} + \frac{(-jn+1)}{(n^2+1)}\,\frac{e^{-jnt}}{2} = \frac{\cos \pi t - \pi \sin \pi t}{1+\pi^2}$$

4) $x(t)$ is real and even

a , d , $\Rightarrow$ true

(b) , (c) $\Rightarrow$ false