Solution.

1.a) As $f_s = 7\text{MHz}$ $f_p = 1.8$ MHz

we have $k = \frac{f_p}{f_s} \approx 0.257$

$$k_1 = \sqrt{\frac{10^{\alpha_p/10}-1}{10^{\alpha_s/10}-1}} = 1.609 \times 10^{-3}$$

thus, $n \geq \frac{-\log k_1}{-\log k} \approx 4.736$ => Arbitrarily $n=5$

$$\Rightarrow C_n = \frac{10^{\alpha_p/10}-1}{(2\pi f_p)^{2n}} = \frac{10^{0.1}-1}{(2\pi \times 1.8 \times 10^6)^{10}} \approx 7.56 \times 10^{-72}$$

b) As loss @ stopband limit states,

$$\alpha_s = 10 \log (1 + K^{2n})$$

$$= 10 \log (1 + C_n \omega_s^{2n})$$

$$= 10 \log \left[1 + 7.56 \times 10^{-72} \times (2\pi \times 7 \times 10^6)^{10}\right]$$

$$= 53.11 \text{ dB}$$

2) $S_k = C_n^{-\frac{1}{2n}} e^{j\pi(n-1+2k)/2n}$

$k = 1, 2, \cdots n$

$$Q_p = \frac{|S_k|}{2\,\mathrm{Re}\{s_k\}}$$

$$|S_k| = C_n^{-\frac{1}{2n}}$$

$$S_k = C_n^{-1/2n}\left[\cos\left(\pi(n-1+2k)/2n\right) + j\sin\left(\frac{\pi(n-1+2k)}{2n}\right)\right]$$

$$Q_p = \frac{\cancel{C_n^{-\frac{1}{2n}}}}{2\cancel{C_n^{-1/2n}}\cos\left(\frac{\pi(n-1+2k)}{2n}\right)}$$

$$Q_p = \frac{1}{2\cos\left(\frac{\pi(n-1+2k)}{2n}\right)}$$

$$Q_{p\,max}\Big|_{k=1 \text{ or } n} = \frac{1}{2\cos\left(\frac{\pi(n+1)}{2n}\right)}$$