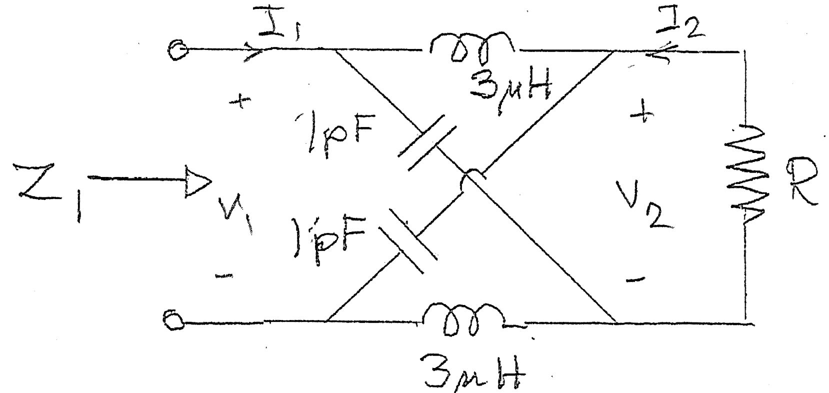
1. In the terminated two-port shown below, the input impedance is Z_1.

a. At what frequencies is $Z_1 = R$ for all values of R?

b. For what value of R does $Z_1 = R$ hold at all frequencies? Why?

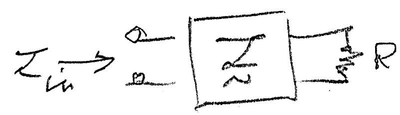


$V_1 = z_{11} I_1 + z_{12} I_2$

$V_2 = z_{21} I_1 + z_{22} I_2$

$V_2 = -I_2 R$

$Z_1 = \frac{V_1}{I_1} = z_{11} - \frac{z_{12} z_{21}}{z_{22} + R}$



1. $z_{11} = \frac{1}{2}\left[\frac{1}{sC} + sL\right] = z_{22}$

$z_{12} = \frac{1}{2}\left[\frac{1}{sC} - sL\right] = z_{21}$ reciprocal

$Z_1 = z_{11} - \frac{z_{12} z_{21}}{z_{22} + R} = R\left(\frac{z_{11} + (L/C)/R}{z_{11} + R}\right)$

a. By inspection, $z_{11} \to \infty$ for $s = 0$ or $s \to \infty$, so $Z_1 \to R$ for $f = 0, \infty$

b. If $(L/C)/R = R$; or $L/C = R^2$,

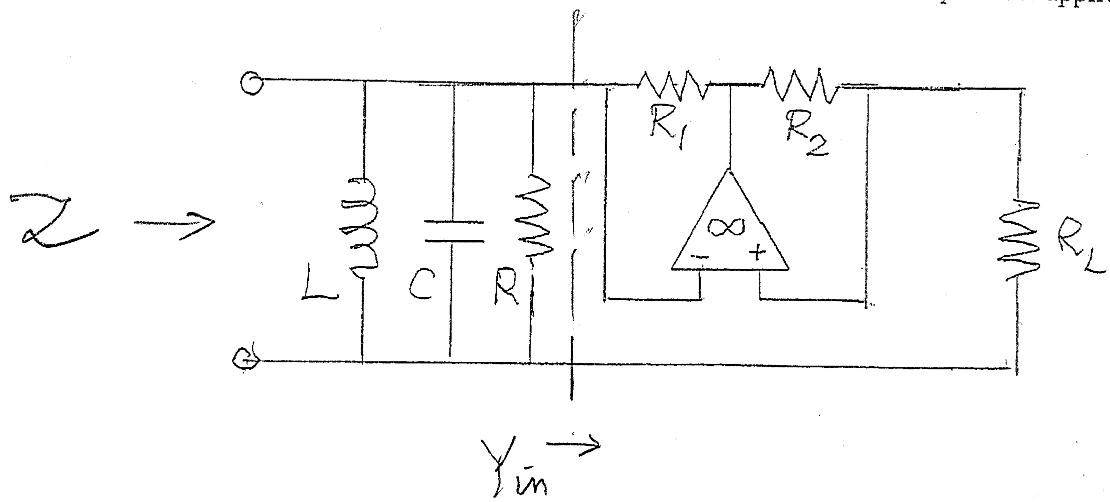
$Z_1(s) \equiv R \;\forall s$. $R \doteq 1.732\ k\Omega$

Or

b. $R = z_{11} - \frac{z_{12}^2}{z_{11} + R}$, $R - z_{11} = -\frac{z_{12}^2}{R + z_{11}}$

$R^2 - z_{11}^2 = -z_{12}^2$, $R^2 = sL \cdot 1/sC = L/C$

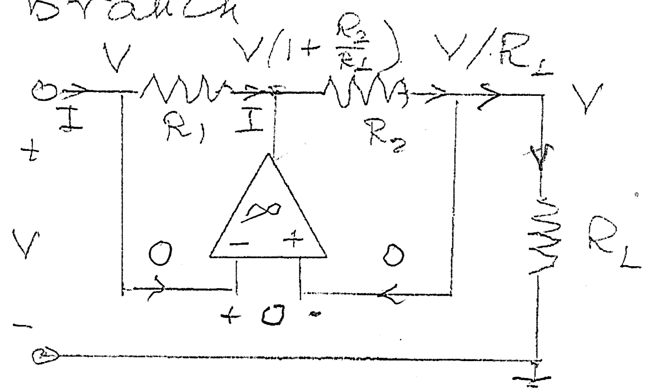
2. What is the input impedance of the circuit shown below? What are its possible applications?



2. The input admittance of the active branch

$$I = -V\frac{R_2}{R_1 R_L}$$

$$Y_{in} = -\frac{R_2}{R_1 R_L}$$



Riordan

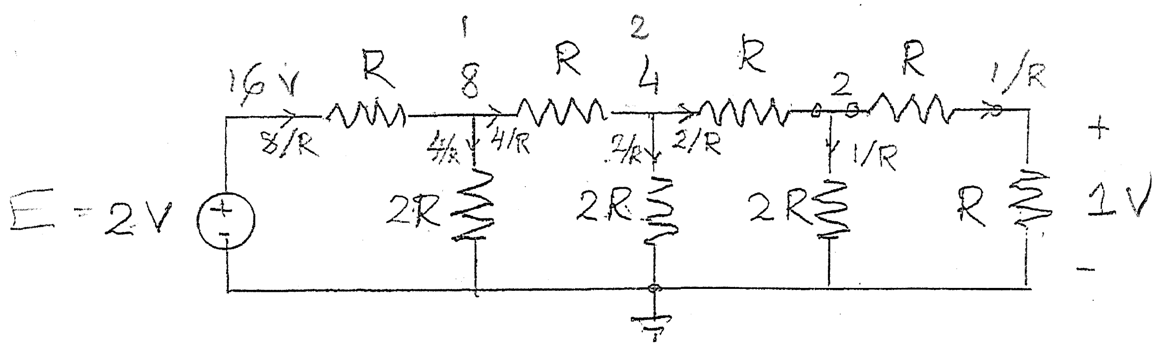
Negative conductance may cancel 1/R → oscillator, high-Q filter

Overall

$$Z_1 = \frac{1}{1/sL + sC + 1/R - R_2/(R_1 R_L)}$$

Q booster, oscillator

3. (Extra credit) Find all node voltages in the circuit shown below.



$1 \rightarrow 2/16 = 1/8$ V

So the node voltages are

2, 1, 0.5, 0.25 and 0.125 V

Or : Resistance after node = 2R

before " " = R

So, at second node, $V_2 = V_1/2 = E/2$,

at 3rd node, $V_3 = E/4$, etc.

R - 2R DAC!