Question 1:
   a) The growth velocity in a chemical vapor deposition reactor is described by the following equation.

   \[ v = \frac{C_g \cdot h_g k_s}{N \left( h_g + k_s \right)} \]

   Where:
   - \( v \) is the growth velocity of the deposited layer.
   - \( k_s \) is the reaction rate on the surface.
   - \( C_g \) is the gas concentration in the chamber.
   - \( h_g \) is the mass transfer coefficient.

   The growth is governed by two processes; mass transport and the reaction rate. The growth rate is limited by the slowest process. In the mass-transport-limited regime we can make the approximation that \( h_g \ll k_s \). Using the following approximation for the mass-transfer coefficient, determine the relationship between the growth velocity, temperature (\( T \)) and velocity of the gas flow (\( u_0 \)) in this regime.

   \[ h_g = \frac{3 \lambda \bar{c}}{4L \sqrt{Re}} \]

   Where:
   - \( h_g \) is the mass transfer coefficient.
   - \( \lambda \) is the mean-free path of the gas molecule.
   - \( \bar{c} \) is the mean gas molecule speed.
   - \( L \) is the length of the substrate being deposited on, in the direction of the gas flow.
   - \( Re \) is the Reynolds Number.

If \( h_g \ll k_s \) then we can make the following approximation:

   \[ \frac{1}{(h_g + k_s)} \approx \frac{1}{k_s} \]

Apply this approximation to our equation for growth velocity:

   \[ v = \frac{C_g \cdot h_g k_s}{N \cdot k_s} = \frac{C_g h_g}{N} \]

Substitute in our equation for mass-transfer coefficient:

   \[ v = \frac{3 \lambda \bar{c} C_g}{4LN \sqrt{Re}} \]
We are interested in the relationship of $v$ with $T$ and $u_0$. So we need to substitute in for all parameters that could depend on either $T$ or $u_0$.

Start with mean free path:

$$\lambda = \frac{1}{\sqrt{2\pi d_0^2 n}}$$

Putting this into the equation for growth velocity:

$$v = \frac{3\bar{c}C_g}{4\sqrt{2\pi d_0^2 nL\bar{N}}} \sqrt{Re}$$

The notation for gas concentration when talking about CVD is $C_g$, and the notation for gas concentration when talking about general gas theory is $n$, but both values are equivalent ($C_g \equiv n$) so we can simplify the growth velocity slightly:

$$v = \frac{3\bar{c}}{4\sqrt{2\pi d_0^2 L\bar{N}}} \sqrt{Re}$$

Now look at the mean particle speed:

$$\bar{c} = \frac{\sqrt{8k_B T}}{\pi m}$$

Substitute this into $v$:

$$v = \frac{8k_B T}{\pi m} \frac{3}{4\sqrt{2\pi d_0^2 L\bar{N}}} \sqrt{Re}$$

Condense the numerical constants:

$$\frac{\sqrt{8} \times 3}{4 \times \sqrt{2}} = \frac{8 \times 9}{16 \times 2} = \frac{9}{4} = \frac{3}{2}$$

So we can write the growth velocity as:

$$v = \frac{3}{2\pi d_0^2 L\bar{N}} \sqrt{\frac{k_B T}{\pi m}} \sqrt{Re}$$

Finally substitute the Reynolds Number:

$$Re = \frac{u_0 \rho L}{\eta}$$
Now there are no other parameters that depend on \( T \) or \( u_0 \), so we can then express the following proportionality relationship:

\[
v \propto T^{1/2} u_0^{1/2}
\]

b) The reaction rate on the surface can be described by the following equation:

\[
k_s = k_0 \exp \left( - \frac{E_A}{k_BT} \right)
\]

Where:
- \( k_s \) is the reaction rate on the surface.
- \( k_0 \) is the reaction rate constant.
- \( E_A \) is activation energy of the reaction.
- \( k_B \) is the Boltzmann constant.
- \( T \) is the temperature.

Determine the relationship between the growth velocity and temperature (\( T \)) in the regime where \( h_g \gg k_s \).

If \( h_g \gg k_s \) then we can make the following approximation:

\[
\frac{1}{(h_g + k_s)} \approx \frac{1}{h_g}
\]

\[
v = \frac{C_g h_g k_s}{N} = \frac{C_g k_s}{N}
\]

Now all we have to is substitute the equation for reaction rate:

\[
v = \frac{C_g h_g k_s}{N} = \frac{C_g k_0 e^{-E_A/k_BT}}{N}
\]

So in this case we have an exponential dependence of growth velocity on temperature.

c) Consider the following low-pressure CVD (LPCVD) reaction:

\[
SiCl_4(G) + 2H_2(G) \rightarrow Si(S) + 4HCl(G)
\]

The following schematic diagram illustrates how one would expect the growth rate to vary as the ratio of the pressure of gases is varied. From this diagram, explain how you would choose the relative pressures of your process gases to grow a polycrystalline film of silicon? [2 marks]
To form polycrystalline silicon, rather than mono-crystalline silicon, the growth velocity should be as high as possible. For this reason a position halfway along the x-axis should be chosen, where the ratio of SiCl₄ is similar to H₂.

Question 2:

a) Using the following equations for particle flux (\(J_A\)), pressure (\(P\)), and mean particle velocity (\(\bar{c}\)),

\[
J_A = \frac{1}{4} n \bar{c} \\
P = nk_B T \\
\bar{c} = \sqrt{\frac{8k_B T}{\pi m}}
\]

Where:
- \(J_A\): particle flux.
- \(n\): particle number density.
- \(\bar{c}\): mean particle velocity.
- \(P\): gas pressure.
- \(k_B\): Boltzmann Constant.
- \(m\): gas particle mass.

show that particle flux can be written as:

\[
J_A = \frac{P}{\sqrt{2\pi mk_B T}}
\]

First re-arrange the Ideal Gas Law in terms of \(n\):

\[
n = \frac{P}{k_B T}
\]

Then substitute \(n\) and \(\bar{c}\) into \(J_A\):

\[
J_A = \frac{1}{4} \frac{P}{k_B T} \sqrt{\frac{8k_B T}{\pi m}}
\]
Start with everything in the square root:

\[ J_A = \frac{P^2 8k_B T}{16k_B^2 T^2 \pi m} \]

Cancel \( T \)'s: and \( k_B \)'s:

\[ J_A = \frac{P^2 T}{2k_B T \pi m} \]

Now take the \( P \) out of the square root:

\[ J_A = \frac{P}{\sqrt{2\pi m k_B T}} \]

b) The equation below gives the fractional coverage of a thin film (\( \theta \)), given a certain adsorption rate (\( k_{ads} \)), desorption rate (\( k_{des} \)), and process gas pressure (\( P \)).

\[ \theta = \frac{KP}{1 + KP}(1 - \exp[-k_{des}(1 + KP)t]) \]

\[ K = \frac{k_{ads}}{k_{des}} \]

Determine what the equation for \( \theta \) reduces to in the below three limiting cases:

a. \( k_{ads} \rightarrow \infty \).

b. \( k_{des} \rightarrow \infty \).

c. \( t \rightarrow \infty \).

a. \( k_{ads} \rightarrow \infty \):

In this limit, was also know:

\[ \lim_{k_{ads} \rightarrow \infty} K = \frac{k_{ads}}{k_{des}} = \infty \]

We can also say the same for \( KP \):

\[ \lim_{K \rightarrow \infty} [KP] = \infty \]

Since \( 1 \ll \infty \), we can also say:

\[ \lim_{KP \rightarrow \infty} \left[ \frac{KP}{1 + KP} \right] = \frac{KP}{KP} = 1 \]

Now look at the exponential term in the limit \( K \rightarrow \infty \):
\[ \lim_{K \to \infty} \exp[-k_{\text{des}}(1 + KP)t] = 0 \]

This technically is only valid for \( t > 0 \), but we will consider non-zero times.

Hence we can then say:

\[ \lim_{k_{\text{ads}} \to \infty} [\theta] = 1 \times (1 - 0) = 1 \]

I.e. for any non-zero time we get complete coverage.

b. \( k_{\text{des}} \to \infty \):

Follow the same steps as before, first determine \( K \) in this limit:

\[ \lim_{k_{\text{des}} \to \infty} \left[ K = \frac{k_{\text{ads}}}{k_{\text{des}}} \right] = 0 \]

Then \( KP \):

\[ \lim_{K \to 0} [KP] = 0 \]

\[ \lim_{KP \to 0} \left[ \frac{KP}{1 + KP} \right] = \frac{0}{1 + 0} = 0 \]

For the exponential, we have to substitute \( K \) in:

\[ \exp[-k_{\text{des}}(1 + KP)t] = \exp\left[-k_{\text{des}}\left(1 + \frac{k_{\text{ads}}}{k_{\text{des}}}P\right)t\right] \]

Multiply out:

\[ \exp[-k_{\text{des}}(1 + KP)t] = \exp[-k_{\text{des}}t - k_{\text{des}}\frac{k_{\text{ads}}}{k_{\text{des}}}Pt] \]

\[ = \exp[-k_{\text{des}}t - k_{\text{ads}}Pt] \]

\[ = \exp[-(k_{\text{des}} + k_{\text{ads}}P)t] \]

Hence for \( t > 0 \):

\[ \lim_{k_{\text{des}} \to \infty} [\exp[-k_{\text{des}}(1 + KP)t] = \exp[-(k_{\text{des}} + k_{\text{ads}}P)t]] = 0 \]

So we can say:

\[ \lim_{k_{\text{des}} \to \infty} [\theta] = 0 \times (1 - 0) = 0 \]

I.e. zero coverage.

c. \( t \to \infty \):
Since in this limit $K$ is finite, we just need to look at the exponential:

$$\lim_{t \to \infty} [\exp(-k_{des}(1 + KP)t)] = 0$$

Therefore:

$$\lim_{t \to \infty} [\theta] = \frac{KP}{1 + KP} \times (1 - 0) = \frac{KP}{1 + KP}$$

c) For a given thin-film growth process, we are told that adatoms on the surface of a substrate will make $\nu_a = 10^{11}$ attempts to desorb per second, and the activation energy for the desorption process is $E_a = 0.45$ eV. The adatoms will also attempt to hop between surface sites (diffuse) with a frequency of: $\nu_d = 10^{14}$ s$^{-1}$. The barrier to hop has an energy of: $E_d = 0.2$ eV, and the distance between sites is 5.5 Å. Determine the diffusion coefficient of adatoms on this surface when held at 300K.

First we need to determine the probability that in one second the adatom will have enough energy to pass over the energetic barrier between sites:

$$P_d = \nu_d e^{-E_d/k_BT}$$

We are given the attempt frequency, energy barrier and temperature, so can just put in the numbers (remembering to convert to Joules):

$$P_d = 10^{14} \times \exp \left[ - \frac{0.2 \times 1.602 \times 10^{-19}}{1.38 \times 10^{-23} \times 300} \right]$$

$$P_d = 4.35 \times 10^{10} \text{s}^{-1}$$

From this probability, and the distance between sites ($l$), we can determine the diffusion coefficient:

$$D = lP_d$$

$$D = (5.5 \times 10^{-10})^2 \times 4.35 \times 10^{10}$$

$$D = 1.32 \times 10^{-8} \text{ m}^2/\text{s}$$

Next we need to determine the probability that an adatom will desorb in one second:

$$P_a = \nu_a e^{-E_a/k_BT}$$

$$P_a = 10^{11} \times \exp \left[ - \frac{0.45 \times 1.602 \times 10^{-19}}{1.38 \times 10^{-23} \times 300} \right]$$

$$P_a = 2.74 \times 10^{3} \text{s}^{-1}$$

We convert this probability / second into a characteristic timescale (lifetime on surface):
\[ \tau_a = \frac{1}{P_a} = 3.65 \times 10^{-4} \text{s} \]

Finally, we can get the diffusion length:

\[ L = \sqrt{D\tau_a} \]

\[ L = \sqrt{1.32 \times 10^{-8} \times 3.65 \times 10^{-4}} = 2.19 \mu\text{m} \]

d) Write We are carrying out growth of a thin film of GaAs on a GaP substrate. The surface energy of GaP is 1.9 J/m², the surface energy of GaAs is 0.86 J/m². The surface energy for the interface between the GaP substrate and the GaAs thin film (\( \gamma_i \)) can be approximated as a function of the GaAs thickness (\( x \)) and the surface stress of the interface (\( \sigma_i \)):

\[ \sigma_i = \frac{2\gamma_i}{x} \]

If we approximate the surface stress as constant at \( \sigma_i = 10^8 \text{ J/m}^3 \), determine the number of monolayers of GaAs we can grow under these conditions, that exhibit layer-by-layer (Frank-van der Merwe) growth. The lattice constant in GaAs is \( a = 5.65 \text{ \AA} \).

The condition for Frank-van der Merwe growth:

\[ \gamma_i + \gamma_f > \gamma_s \]

The transition from Frank-van der Merwe to Volmer-Weber growth therefore occurs when:

\[ \gamma_i + \gamma_f = \gamma_s \]

Substitute in the surface energy for the interface:

\[ \gamma_i = \frac{x\sigma_i}{2} \]

\[ \frac{x\sigma_i}{2} + \gamma_f = \gamma_s \]

Re-arrange in terms of the film thickness:

\[ \frac{x\sigma_i}{2} = \gamma_s - \gamma_f \]

\[ x = \frac{2}{\sigma_i}(\gamma_s - \gamma_f) \]

Now just put in the numbers:

\[ x = \frac{2}{10^8}(1.9 - 0.86) \]

\[ x = 20.8 \text{ nm} \]

Now we just need to determine how many monolayers of GaAs there are in 20.8nm. We are told the lattice constant is \( a = 5.65 \text{\AA} \), hence the number of monolayers:
\[
\frac{208}{5.65} = 36.8
\]

So, if we are talking about integer monolayers, it would take 37ML for the growth to become Volmer-Weber.

e) Describe what is meant by:
   a. An amorphous film.
   b. A polycrystalline film.
   c. A single-crystal film.
      a. A fully disordered film with no significant order.
      b. Randomly orientated grains of crystalline material, separated by boundaries.
      c. Complete long range. A perfect crystal.

f) Describe a technique to measure the thickness of a thin-film.

There are a few potential answers to this question. Techniques include
   - Profilometry.
   - Four-point Probe.
   - Ellipsometry.
   - Reflection high-energy electron diffraction (for MBE growth).

See Lecture 12, slides 70-76 for descriptions.

Question 3:
   a) Explain why reducing resolution is important for integrated circuit design.

   The processing power of integrated circuits depends on the number of transistors in the chip. The ability to reduce the feature size of transistors, and increase the density of transistors, is hence vital to improving the performance of CPUs.

   b) Briefly describe the spin-coating procedure.

   1. Photoresist (in solvent) is deposited in center of the wafer. Wafer is held in place with a vacuum chuck.
   2. Wafer is rotated slowly (200 rpm) to distribute material.
   3. Accelerate the wafer to final speed (~5000 rpm). Spin the wafer at constant speed for 30 - 60s to evaporate the solvent.
   4. A uniform film is formed.

   c) We are exposing a wafer to 365nm light through a 200 nm aperture, at a distance of 10µm from the wafer surface. What approximation for diffraction is most appropriate in this case: Fresnel or Fraunhofer?

   To answer this question we need to evaluate the Fresnel number (\(F\)):

   \[
   F = \frac{W^2}{L\lambda}
   \]

   Where:
- $W$ is the size of the aperture.
- $L$ is the distance of the wafer from the aperture.
- $\lambda$ is the wavelength of the incident light.

For $F \ll 1$ the diffraction is described by Fraunhofer diffraction. For $F \gg 1$ the diffraction is described by Fresnel diffraction.

Evaluating $F$ (in nm):

$$F = \frac{200^2}{10^4 \times 365} = 0.011$$

Hence this situation is best described by Fraunhofer diffraction.

d) We are carrying out a simple photolithography exposure procedure using proximity printing. If the mask is $L = 50\mu m$ from the wafer surface, the wavelength of light is $\lambda = 193\text{nm}$, and the experimental parameter $k$ is 1, determine the minimum resolvable feature size.

For Fresnel diffraction (how we describe proximity printing), the minimum resolvable feature size is described by:

$$W_{\text{min}} = \sqrt{kL\lambda}$$

Putting the numbers above in (in nm):

$$W_{\text{min}} = \sqrt{1 \times 5 \times 10^4 \times 193} = 3.11\mu m$$

e) In an air-based projection printing system the focal length ($f$) is $4 \times$ the diameter of the lens ($d$), the wavelength is $\lambda = 157\text{nm}$ and the experimental parameters $k_1$ and $k_2$ are both 0.3. Determine the feature size and the depth of field in this system.

The resolution for projection printing (Fraunhofer) is described by the Rayleigh Criterion ($R$):

$$R = k_1\lambda f \frac{d}{d^2}$$

Put the numbers in (in nm):

$$R = 0.3 \times 157 \times 4 = 188\text{ nm}$$

The depth of focus ($\delta$) is given by:

$$\delta = k_2\lambda \left( \frac{f}{d} \right)^2$$

Putting the numbers in (nm):

$$\delta = 0.3 \times 157 \times 4^2 = 754\text{ nm}$$
f) Describe how standing waves can be a problem in photolithography and an approach to overcome the problem.

When the surface exposed to light is reflective (e.g. certain metals), light can be back-scattered from the surface. Interference between the incoming and reflected coherent light can result in standing wave patterns, leading to intensity variations with depth in the photoresist. This can result in wave-like features being created in the material being etched (see slide 54 of Lecture 13 for examples).

To suppress this problem, antireflective coatings (ARCs) can be applied to the metal surfaces prior to photoresist deposition, then removed with the photoresist in development.

Question 4:
   a) Describe how a phase-shift mask can be utilized to improve resolution and contrast in photolithography.

   Phase-shift masks take advantage of the amplitude of light. Since the amplitude of light can be positive or negative (whereas intensity can only be positive), by making adjacent diffraction peaks 180° (or $\pi$ in radians) out of phase these adjacent diffraction patterns destructively interfere, producing a pattern with better resolution and contrast.

   This achieved in practice by constructing a mask with alternating apertures having a high refractive index material inform of the opening. By having a higher refractive index, light is slowed down as it passes through the medium, shifting its phase relative to those regions not covered by this medium.

   b) Describe how multilayer resist processing can be used to improve the MTF in a photolithography process.

   This multilayer resist process involves the use of a contrast enhancement layer (CEL) on top of the conventional photoresist. The CEL is a photo-bleachable material, meaning it will change its optical transmittance on exposure to light. By illuminating the CEL with light of the correct wavelength (using a conventional projection lithography technique) patterned regions will become transparent to the UV light used in processing the conventional photoresist below.

   The advantage of carrying out this multilayer resist process is that, we now have a contact mask on the surface of the photoresist, with the same feature size as would be obtainable using conventional projection printing. Because the CEL is typically an organic polymer (e.g. Poly(methyl methacrylate) – PMMA) this is a contact printing technique that does not share the same issues with conventional contact printing, i.e. metal contamination on wafer. The CEL can be removed using normal development techniques, such as piranha etching.

   c) Determine the wavelength of electrons accelerated using a 150 kV source.

   The wavelength of massive particles is given by the de Broglie Equation. For high-energy particles (those that travel close to the speed of light), we must use the relativistic de Broglie Equation. If you are uncertain, always use the relativistic equation as it is correct for all particle energies:
\[ \lambda = \frac{h}{\sqrt{2m_0eV} \sqrt{1 + \frac{eV}{2m_0c^2}}} \]

Where:
- \( \lambda \) is the particle wavelength.
- \( h \) is the Planck Constant.
- \( e \) is the charge of the particle.
- \( V \) is the accelerating voltage.
- \( m_0 \) is the rest mass of the particle.
- \( c \) is the speed of light in a vacuum.

This is simply an exercise in entering numbers. There are plenty of constants here, so we work in SI:

\[ \lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.11 \times 10^{-31} \times 1.602 \times 10^{-19} \times 150 \times 10^{3}}} \sqrt{1 + \frac{1.602 \times 10^{-19} \times 150 \times 10^{3}}{2 \times 9.11 \times 10^{-31} \times 3.0 \times 10^{8}}} \]

\[ \lambda = 2.96 \text{ pm} \]

For interest, the non-relativistic value is:

\[ \lambda = 3.17 \text{ pm} \]

**Question 5:**

a) Below is a diagram of an etch process. We are told the vertical etch rate in the film is \( R_f = 80 \text{ nm/min} \), the film-mask selectivity is \( S_{fm} = 5 \), and the film-substrate selectivity is \( S_{fs} = 10 \). If we carry out an etch this etch for a period of 3 minutes, determine the remaining mask height (\( h_m \)) and the depth of the substrate we have etched (\( h_s \)). For this etch assume the anisotropy is \( A = 1 \).

Since \( A = 1 \), we are here just talking about vertical etching. We will first consider the photoresist (mask). We are told the vertical etch rate of the film is \( R_f = 80 \text{ nm/min} \) and the film-mask selectivity is \( S_{fm} = 5 \). The definition of film-mask selectivity is:
\[ S_{fm} = \frac{R_f}{R_m} \]

Where \( R_m \) is the vertical etch rate of the mask. Hence:

\[ R_m = \frac{R_f}{S_{fm}} = \frac{80}{5} = 16 \text{ nm/min} \]

We know the etch time is 3 minutes, so the thickness of the mask etched is:

\[ \Delta h_m = 16 \times 3 = 48 \text{ nm} \]

The starting thickness of the etch is 100nm, so the final thickness is therefore:

\[ h_m = 100 - \Delta h_m = 100 - 48 = 52 \text{ nm} \]

To determine the amount of the substrate etched is to first calculate the amount of time required to etch through the film, and hence expose the substrate. The film is 200 nm thick, therefore the time to etch the film is:

\[ t_f = \frac{200}{R_f} = \frac{200}{80} = 2.5 \text{ min} \]

Hence the substrate is exposed for \( t_s = 0.5 \) min. We are given the film-substrate selectivity as \( S_{fs} = 10 \). Hence we can determine the vertical etch rate in the substrate \( (R_s) \):

\[ S_{fs} = \frac{R_f}{R_s} \]

\[ R_s = \frac{R_f}{S_{fs}} = \frac{80}{10} = 8 \text{ nm/min} \]

The thickness of substrate etched is then:

\[ h_s = R_s t_s = 8 \times 0.5 = 4 \text{ nm} \]

b) Tetrafluoromethane (CF\(_4\)) is a process gas routinely employed in silicon-based VLSI dry etching. Neutral CF\(_4\) is not hugely useful as an etchant, but its reaction products are. Describe the role of 3 products of CF\(_4\), and what they are typically used to etch.

1. In the plasma charged CF\(_4^+\) ions are formed. These ions can be accelerated towards a surface and used for a highly-anisotropic, but not largely selective etchant for any surface,
2. Neutral CF\(_2\) can also be dissociated from neutral CF\(_4\), in a plasma chamber. Neutral CF\(_2\) does not react to form gas products with common photoresists, but does produce gas products from reacting with silicon oxide:

\[ CF_2(G) + SiO_2(S) \rightarrow CO_2(G) + SiF_2(G) \]

Neutral CF\(_2\) will also react with the silicon substrate, but the solid product is carbon, meaning it must be cleaned with piranha solution if etched.
$$CF_2(G) + Si(S) \rightarrow C(S) + SiF_2(G)$$

3. Finally, elemental fluorine free radicals can also form in a CF₄ plasma. Elemental fluorine can etch the silicon substrate, in a similar manner to an HF etch:

$$2F(G) + Si(S) \rightarrow SiF_2$$

c) Describe the concepts of RIE lag and over-etch.

In reactive ion etching (RIE) it is often observed that wider features are etched more quickly than narrow features. The reasons for this are numerous and outside the scope of the course. However it is a noteworthy phenomenon and is termed RIE lag.

Over-etch is simply the process of etching until all features are etched to the target depth. If smaller features take longer, this will mean that some larger features will be etched for longer than necessary, and could lead to some unintentional etching of the layer below.

d) Describe a buffered hydrofluoric acid etch.

Hydrofluoric acid is used to etch silicon. Sometimes ammonium fluoride (NH₄F) is mixed with hydrofluoric acid (HF), in what is called a buffered HF etch. This serves two purposes. Firstly it allows the etch rate to be more carefully controlled through the composition of HF:NH₄F. Secondly it allows the etch to remain stable for longer by slowing down the depletion of F.

e) The figure below shows an SiO₂ surface patterned with a photoresist mask. The photoresist has a slope of $\theta=30^\circ$. We carry out an etch of the SiO₂ for 2 minutes. The process we employ has an anisotropy of 0.25 and a vertical etch rate of SiO₂ of $R_{v, SiO_2} = 400$nm/ min. If the film-mask selectivity is $S_{fm} = S_{SiO_2, resist} = 10$, determine the width of the region in the SiO₂ which is not flat (i.e. the region labeled $x$ in Figure 2(b)). Assume the SiO₂ is initially perfectly flat, the photoresist is sufficiently thick that it does not get depleted during the 2 minute etch, and assume the anisotropy is the same for both the SiO₂ and the resist.

The key to this question is to understand that we are interested in the lateral distance etched of the photoresist, not the SiO₂. To understand why that is the case, consider the SiO₂ surface, which is initially flat. If there were no lateral etching of the photoresist (or indeed, no
photoresist at all), the SiO\(_2\) would be etched uniformly, regardless of the anisotropy, as no sides of the SiO\(_2\) are exposed to be etched laterally. However, if the photoresist changes its lateral coverage over time, this will mean that different parts of the SiO\(_2\) surface will be exposed at different times in the process. This essentially means that the regions under the photoresist that are removed are etched for different periods of time. At the start of the etch, the SiO\(_2\) under the very right of the photoresist in Figure 2(a) will be exposed immediately, and have the same etch time as the rest of the SiO\(_2\) exposed. At the end of the etch, the SiO\(_2\) under the very right photoresist in Figure 2(b) will have only just been exposed, and hence have an etch time of zero. Between these two extremes, the SiO\(_2\) will be exposed for varying amount of time, depending on when the photoresist is completely removed.

So to answer the question, all we need to do is determine the length of the photoresist that has been removed during the process, as any SiO\(_2\) under this region will have a slope. To do this we use our equation for etching a feature with a slope:

\[
x = \left( \frac{R_v}{\tan \theta} + R_l \right) t
\]

As we are interested in etching the resist, these etch rates refer to the etch rates in the resist:

\[
R_{v,m} = R_v \\
R_{l,m} = R_l
\]

Where the second subscript \(m\) refers to the mask. We are told the vertical etch rate in the SiO\(_2\) is \(R_{v,f} = 400\)nm / min (\(f\) refers to the film = SiO\(_2\)). We are also told the film-mask selectivity is: \(S_{fm} = 10\). Hence the vertical etch rate in the mask (resist) is:

\[
R_{v,m} = \frac{R_{v,f}}{S_{fm}} = \frac{400}{10} = 40\text{nm/min}
\]

We can get the lateral etch rate from the vertical etch rates in the mask using the anisotropy parameter:

\[
A = 1 - \frac{R_{l,m}}{R_{v,m}}
\]

We are told in the question the anisotropy is \(A = 0.25\), and we need to determine the lateral etch rate:

\[
AR_{v,m} = R_{v,m} - R_{l,m}
\]

\[
R_{l,m} = R_{v,m} - AR_{v,m}
\]

\[
R_{l,m} = R_{v,m}(1 - A)
\]

Putting in the numbers:
\[ R_{v,m} = 40(1 - 0.25) \]
\[ R_{l,m} = 40 \times 0.75 = 30 \text{nm/min} \]

We now have the vertical etch rate in the resist \( R_{v,m} = 40 \text{nm/min} \), the lateral etch rate in the resist \( R_{l,m} = 30 \text{nm/min} \), the slope of the resist \( \theta = 30^\circ \) and the etch time is \( t = 2 \text{ minutes} \). So we can finally put the numbers into our equation for etching a sloped feature:

\[
x = \left( \frac{R_v}{\tan \theta} + R_l \right) t = \left( \frac{R_{v,m}}{\tan \theta} + R_{l,m} \right) t
\]
\[ x = \left( \frac{40}{\tan 30^\circ} + 30 \right) \times 2 \]

Make sure you have your calculator set to degrees, or convert 30 degrees to radians if you want to remain in radians.

\[ x = \left( \frac{40}{0.577} + 30 \right) \times 2 = 198.6 \text{ nm} \]

**Question 6:**

a) State two reasons why Moore’s Law cannot continue indefinitely.

Any two of the following (or any other reasonable answer):
- Power consumption is becoming too high
- It is too expensive to pattern smaller features.
- Wavefunctions will leak into adjacent components.
- Atoms have a finite size.

b) Briefly explain the operating principle of electronic devices based on phase-change materials.

Phase change materials are those which are observed to undergo an electronic phase transition under an applied external stimulus (e.g. heat). There are certain materials (e.g. VO\(_2\)) which exhibit a conducting-to-insulating phase transition close to room temperature.

To make an electronic device from such a material one would have to induce the phase transition from an electrical signal. For example, one could use a piezoelectric material to induce strain which could in-turn induce a transition between conducting and insulating states.

This technology is potentially valuable because it would enable discontinuous switching between on and off states at certain applied voltages, rather than a gradual increase in conductance as observed in traditional field-effect transistors.

c) State a problem that could be faced when using graphene in the place of silicon in microelectronics.

Graphene has no band gap so it is challenging to turn it off. It is essentially a conductor not a semiconductor.
d) Briefly explain how a cross-bar array of programmable resistors could be used to efficiently carry out the multiplication of a matrix and a column vector.

If one has a matrix of weights \( w_{ij} \) that need to be applied to a column vector \( x_i \) the operation would look something like the following:

\[
y_j = \sum_{i=1}^{n} w_{ij} x_i
\]

To carry this operation on a conventional von Neumann architecture one would have to store the weights as floating point variables. This would require them to be digitized to 1’s and 0’s using whatever algorithm is most appropriate, then operated on.

If one were equivalently able to set the conductance \( G_{ij} \) of a two-dimensional array or programmable resistors, then the same operation could be carried out much more quickly by applying a voltage across each element and simply using Ohm’s Law:

\[
l_j = \sum_{i=1}^{n} G_{ij} x_i
\]

e) In RSA encryption, a private key and a public key are used to encrypt data. If 128-bit encryption was used, the number of possible values the private key could take would be roughly \( 3.4 \times 10^{34} \). Suppose a quantum computer was constructed that was able to encode then check a number of variables against the private key. Without knowing anything about the architecture of the quantum computer, approximate how many qubits would be required to crack the encryption in a single set of sequential operations?

This question omits a lot of detail, so all it is really asking is how many qubits are required to encode \( 3.4 \times 10^{34} \) variables.

From Lecture 16 we saw that the number of variables one can encrypt and process depends exponentially on the number \( n \) of qubits as \( 2^n \). If we need to check roughly \( 3.4 \times 10^{34} \) values of the private key. I.e.:

\[
2^n = 3.4 \times 10^{34}
\]

We can evaluate \( n \) by taking the base-2 logarithm:

\[
\log_2[2^n] = \log_2[3.4 \times 10^{34}]
\]

\[
n = \log_2[3.4 \times 10^{34}] = 114.71
\]

I.e. we should round up so the operation is possible:

\[
n = 115
\]

f) What is meant by the term “quantum supremacy”? 

The demonstration that a quantum computer can solve a problem that a classical computer cannot in a reasonable amount of time (e.g. the lifetime of the universe).