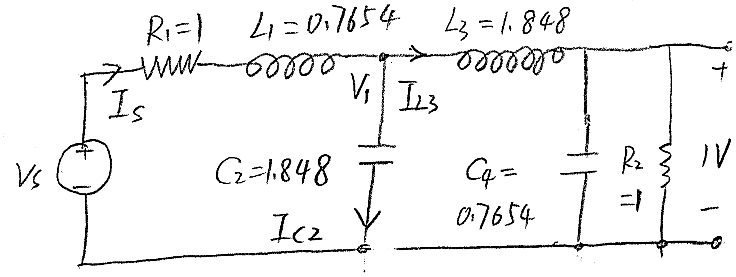
HW2 Solution



1. Assume $V_{out}=1V$.

$$I_{L3}=\left(SC_4+\frac{1}{R_2}\right)\cdot V_{out}=SC_4+\frac{1}{R_2}$$

$$V_1=V_{out}+I_{L3}\cdot SL_3=1+\left(SC_4+\frac{1}{R_2}\right)\cdot SL_3$$

$$I_{C2}=SC_2\cdot V_1=SC_2\left[1+\left(SC_4+\frac{1}{R_2}\right)\cdot SL_3\right]$$

$$I_S=I_{L3}+I_{C2}=\left(SC_4+\frac{1}{R_2}\right)+SC_2\left[1+\left(SC_4+\frac{1}{R_2}\right)\cdot SL_3\right]$$

$$V_S=I_S(R_1+SL_1)+V_1=\left[\left(SC_4+\frac{1}{R_2}\right)+SC_2\left[1+\left(SC_4+\frac{1}{R_2}\right)\cdot SL_3\right]\right](R_1+SL_1)+$$

$$+\ 1+\left(SC_4+\frac{1}{R_2}\right)\cdot SL_3$$

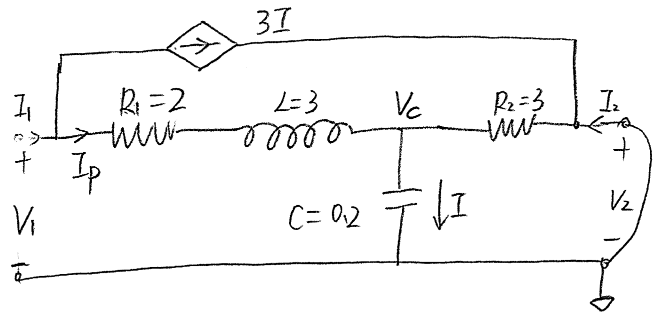
$$H(S)=\frac{V_{out}}{V_S}=\frac{1}{V_S}=\frac{1}{2(\boxed{S^4}+2.6135\boxed{S^3}+3.4149\boxed{S^2}+2.6135\boxed{S}+1)}$$

$$H(jw)=\frac{1}{2(W^4-2.6135jw^3-3.4149w^2+2.6135jw+1)}$$

2. $Y_{11}=\frac{I_1}{V_1}\Big|_{V_2=0}$ $\quad Y_{12}=\frac{I_1}{V_2}\Big|_{V_1=0}$

$Y_{21}=\frac{I_2}{V_1}\Big|_{V_2=0}$ $\quad Y_{22}=\frac{I_2}{V_2}\Big|_{V_1=0}$

Y_{11} and Y_{21}, the network becomes :



$$V_C = I \cdot \frac{1}{sC} \qquad I_p = I + \frac{V_C}{R_2} = I\left(1 + \frac{1}{sCR_2}\right)$$

$$V_1 = I_p (R_1 + sL) + V_C$$

$$I_1 = I_p + 3I$$

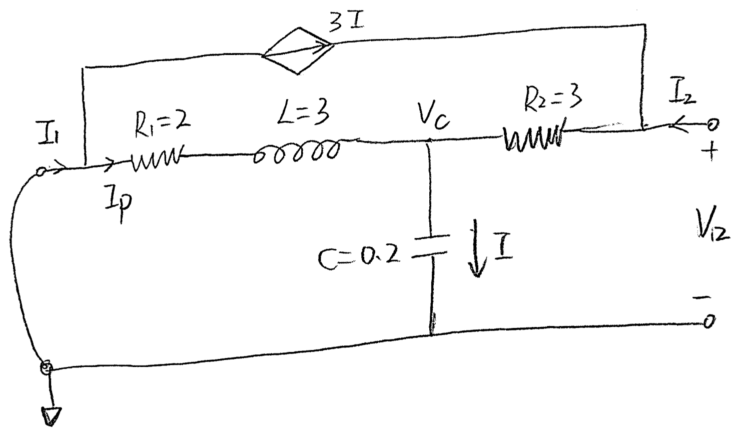
$$Y_{11} = \frac{I_1}{V_1} = \frac{1 + 4sR_2C}{(sR_2C + 1)(R_1 + sL) + R_2} = \frac{1 + 2.4\boxed{s}}{1.8\boxed{s^2} + 4.2\boxed{s} + 5}$$

$$Y_{21} = \frac{I_2}{V_1}, \qquad V_1 = I_p (R_1 + sL) + V_C$$

$$I_2 = -3I - \frac{V_C}{R_2}$$

$$\Rightarrow Y_{21} = -\frac{1 + 3sR_2C}{(sR_2C + 1)(R_1 + sL) + R_2} = \frac{-1.8\boxed{s} - 1}{1.8\boxed{s^2} + 4.2\boxed{s} + 5}$$

to calculate Y_{12} and Y_{22}



$$V_C = I \cdot \frac{1}{sC} \qquad I_p = -\frac{V_C}{R_1+sL} \qquad I_1 = I_p + 3I$$

$$V_2 = (I - I_p)\cdot R_2 + V_C$$

$$\Rightarrow Y_{12} = \frac{I_1}{V_2} = \frac{3sC(R_1+sL)-1}{R_2 + sCR_2(R_1+sL) + R_1 + sL}$$

$$= \frac{1.8\boxed{s^2} + 1.2\boxed{s} - 1}{1.8\boxed{s^2} + 4.2\boxed{s} + 5}$$

Y_{22}:

$$V_2 = (I - I_p)\cdot R_2 + V_C$$

$$I_2 = I - I_p - 3I = -2I - I_p$$

$$\Rightarrow Y_{22} = \frac{I_2}{V_2} = \frac{-2sC(R_1+sL)+1}{R_2 + sCR_2(R_1+sL) + R_1 + sL}$$

$$= \frac{-1.2\boxed{s^2} - 0.8\boxed{s} + 1}{1.8\boxed{s^2} + 4.2\boxed{s} + 5}$$

$$Y_{11} = \frac{2.4\boxed{S} + 1}{1.8\boxed{S^2} + 4.2\boxed{S} + 5}$$

$$Y_{21} = \frac{-1.8\boxed{S} - 1}{1.8\boxed{S^2} + 4.2\boxed{S} + 5}$$

$$Y_{12} = \frac{1.8\boxed{S^2} + 1.2\boxed{S} - 1}{1.8\boxed{S^2} + 4.2\boxed{S} + 5}$$

$$Y_{22} = \frac{-1.2\boxed{S^2} - 0.8\boxed{S} + 1}{1.8\boxed{S^2} + 4.2\boxed{S} + 5}$$