1. (Arithmetic Coding).
Let \( P(a_1) = 0.2, P(a_2) = 0.3, \) and \( P(a_3) = 0.5 \). Find the intervals that represents the arithmetic code, and give a valid arithmetic codeword for the following sequences

(a) \( a_1a_1a_3a_2a_3a_1 \)
(b) \( a_1a_1a_3a_3a_2 \)

2. (LZ77). Suppose the sequence to be encoded is 
\( cabracadabrarrarrad. \)
Suppose the length of the window is 13 characters of which 7 characters are used for the search buffer and 6 characters are used of the look-ahead buffer. The current condition is \( cabraca|dabra| \) where the characters to the left of “|” are in the search buffer while the characters to the right of | are in the look-ahead buffer. Find the triplets (offset, length, next symbol) produced the LZ77 compression for the sequence.

3. (DCT).
(a) From the definition of 2-D DCT, show that the 2-D DCT of pixels values in an \( N \times N \) image can be accomplished by first taking the 1-D DCT along the rows of the image (\( N \) points), then another 1-D DCT along the columns (\( N \) points) on the previous results of the row DCT.
(b) Find the 2-D DCT of the \( 4 \times 4 \) image
\[
\begin{pmatrix}
0 & 255 & 0 & 255 \\
0 & 255 & 0 & 255 \\
0 & 255 & 0 & 255 \\
0 & 255 & 0 & 255 \\
\end{pmatrix}
\]
Comment on the results.

4. (JPEG). Suppose the coefficients of an \( 8 \times 8 \) image are:
\[
\begin{pmatrix}
35 & 12 & 0 & 0 & 0 & 0 & 0 & 0 \\
21 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-23 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}
\]
(a) Does the block likely to have more variations in the vertical or horizontal directions?
(b) Determine the runlength encoding pairs for the AC coefficients, then determine the actual coded bits for the first 3 RLE pairs.