Derivation of SNR subject to quantization

\[ \text{SNR} = 10 \log_{10} \frac{P_{\text{signal}}}{P_{\text{noise}}} \]

\[ P_{\text{signal}} = \int_{-\infty}^{\infty} x^2 f(x) \, dx = 2^2 \]

\[ P_{\text{noise}} = \int_{-\infty}^{\infty} e^2 f(e) \, de \]

(assume zero mean)

1. Assume that \( e[n] = x_q[n] - x[n] \), where \( e[n] \) is error signal, \( x_q[n] \) is a quantized version of \( x[n] \).

2. Assume that \( e[n] \) is i.i.d and uniformly distributed between \( -\frac{\Delta}{2} < e[n] < \frac{\Delta}{2} \), where \( \Delta \) is the quantization interval. Then:

\[ \text{var}(e) = P_{\text{noise}} = \int_{-\infty}^{\infty} e^2 f(e) \, de = \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} \frac{e^2}{\Delta} \, de = \frac{\Delta^2}{12} \]

Now, \( \Delta = \frac{X_{\text{max}}}{2^{N-1}} \), where \( N \) is the number of bits used in quantization (see picture), so

\[ P_{\text{noise}} = \frac{X_{\text{max}}^2}{2} \]

(12)
\[
\frac{P_{\text{signal}}}{P_{\text{noise}}} = \frac{2x^2}{\sigma_{\text{noise}}^2} = \frac{2x^2}{x_{\text{max}}^2} (12) \left( 2^{2(N-1)} \right)
\]

\[\text{SNR} = 10 \log_{10} \frac{P_{\text{signal}}}{P_{\text{noise}}} = 20 \log_{10} \frac{3x}{10 X_{\text{max}}} + 10 \log_{10} 12 - 20 \log_{10} 10 + 20 N \log_{10} 2\]

Now since \[20 \log_{10} 2 \approx 6.02 \text{ dB}\]

Each bit adds roughly about 6.02 dB to the SNR (look at the term \[-20 \log_{10} 2\])

\[N \times 20 \log_{10} 2\]