**Problem 9, Page 504**

(a) The probability that any node transmits successfully in a given slot, \( E(p) \)

\[
E(p) = Np(1-p)^{2(N-1)}
\]

\[
E'(p) = \frac{dE(p)}{dp} = N(1-p)^{2(N-1)} + Np(-1)(2N-2)(1-p)^{2N-3}
\]

\[
E'(p) = 0 \implies p = \frac{1}{2N-1} \equiv p^*
\]

(b) Efficiency of pure ALOHA

\[
E(p^*) = \frac{N}{2N-1}(1 - \frac{1}{2N-1})^{2N-2} = \left\{ \frac{N}{2N-1} \right\} \left\{ (1 - \frac{1}{2N-1})^{2N-1} \right\} \left\{ (1 - \frac{1}{2N-1})^{-1} \right\}
\]

Since

\[
\lim_{N \to \infty} \frac{N}{2N-1} = 1/2,
\]

\[
\lim_{N \to \infty} (1 - \frac{1}{2N-1})^{2N-1} = 1/e \quad \text{(from Hint given in previous problem, Problem 7)}
\]

and

\[
\lim_{N \to \infty} (1 - \frac{1}{2N-1})^{-1} = \lim_{N \to \infty} (1/(1 - \frac{1}{2N-1})) = 1,
\]

then

\[
\lim_{N \to \infty} E(p^*) = \frac{1}{2e}
\]

**Problem 10, Page 504**

(a) A’s average throughput is given by \( p_A(1 - p_B) \).

Total efficiency is \( p_A(1 - p_B) + p_B(1 - p_A) \).

(b) A’s throughput is \( p_A(1 - p_B) = 2p_B(1 - p_B) = 2p_B - 2(p_B)^2 \).

B’s throughput is \( p_B(1 - p_A) = p_B(1 - 2p - B) = p_B - 2(p_B)^2 \).

Clearly, A’s throughput is not twice as large as B’s.

In order to make \( p_A(1 - p_B) = 2p_B(1 - p_A) \), we need that \( p_A = 2(p_A/p_B) \).

(c) A’s throughput is \( 2p(1 - p)^{N-1} \), and any other node has throughput \( p(1 - p)^{N-2}(1 - 2p) \).

**Problem 13, Page 505**

The length of a polling round is \( N(Q + R + d_{poll}) \).

The number of bits transmitted in a polling round is \( NQ \). Hence, the maximum throughput is

\[
\frac{NQ}{N(Q/R + d_{poll})} = \frac{R}{1 + d_{poll}R/Q}
\]

**Problem 14, Page 505**

(a) and (b), see figure below.

(c) 1. Forwarding table in E determines that the datagram should be routed to interface 192.168.3.002.
2. The adapter in E creates an Ethernet frame with Ethernet destination address 88-88-88-88-88-88.
3. Router 2 receives the frame and extracts the datagram. The forwarding table in this router indicates that the datagram is to be routed to 198.168.2.002.
4. Router 2 then sends the Ethernet frame with the destination address of 33-33-33-33-33-33 and source address of 55-55-55-55-55-55 via its interface with IP address of 198.168.2.003.
5. The process continues until the packet reaches Host B

(d) E knows (via its forwarding table) that it has go through Router 1 (Interface IP 192.168.3.002) to reach B. So, E
must rely on its ARP to determine the MAC address of 198.168.3.002. Host E sends out an ARP query packet within a broadcast Ethernet frame. Router 2 receives the query packet and recognizes its IP address; so it sends to Host E an ARP response packet, informing it of its MAC address. This ARP response packet is carried by an Ethernet frame with Ethernet destination address 77-77-77-77-77-77, so only E will grab it. From now on, E follows the steps given in c) above to send its frame to B.

Problem 20, Page 507

a) Let \( p_s \) be the probability that a given time slot contains a successful transmission. We have already derived \( p_s \) in class, which is
\[
 p_s = Np(1-p)^{N-1}.
\]

Now, let \( Y \) be a random variable representing the number of consecutive unproductive slots until a success. We can write
\[
P[Y = 0] = p_s, \quad P[Y = 1] = p_s(1-p_s), \quad P[Y = 2] = p_s(1-p_s)^2, \quad \text{etc.}
\]
That is,
\[
P[Y = m] = p_s(1-p_s)^m
\]
for all \( m = 0, 1, \ldots \)

Thus, the average number of consecutive unproductive slots, \( x \), is the expectation/average of \( Y \), which is by definition equal to \( x = E[Y] = \sum_{m=0}^{\infty} m p_s(1-p_s)^m = \frac{1-p_s}{p_s} \).

Hence,
\[
x = \frac{1-Np(1-p)^{N-1}}{Np(1-p)^N} \quad \text{and efficiency} = \frac{k}{k+2} = \frac{k}{k+1-Np(1-p)^N}.
\]

b) Maximizing efficiency is equivalent to minimizing \( x \), which is in turn equivalent to maximizing \( p_s \). We know from class that \( p_s \) is maximized for \( p_s^* = 1/N \).

c) efficiency = \[
 \frac{k}{k+2} = \frac{k}{k+1-Np(1-p)^N} \quad \text{and} \quad \lim_{N \to \infty} \frac{k}{k+1-Np(1-p)^N} = \frac{k}{k+e-1}
\]

d) It is clear that \( \frac{k}{k+e-1} \) goes to 1 as \( k \) to infinity.

Problem 21, Page 507

i) from A to left router:
Source MAC address: 00-00-00-00-00-00
Destination MAC address: 22-22-22-22-22-22
Source IP: 111.111.111.001
Destination IP: 133.333.333.003
Problem A
We need to make sure that one end of the Ethernet is able to detect the collision before it completes its transmission of a frame. Thus, a minimum frame size is required.

Let BW denote the bandwidth of the Ethernet. Consider the worst case for Ethernet’s collision detection:
1. At \( t=0 \) (bit times): A sends a frame
2. At \( t=d_{prop}^{-1} \) (bit times): B sends a frame right before it can sense A’s first bit.
3. At \( t=2d_{prop}^{-2} \) (bit times): If A finishes its transmission of its last bit just before B’s frame arrives at A, then, A won’t be able to detect a collision before finishing its transmission of a frame. Thus, in order for A to detect collision before finishing transmission, the minimum required frame size should be \( \geq 2d_{prop}^{-1} \) (bit times).

Assume that the signal propagation speed in 10BASE-T Ethernet is \( 1.8 \times 10^8 \text{m/sec} \).
As \( d_{prop} = d/(1.8 \times 10^8) \times BW \) (here, we need to convert the propagation delay in seconds to bit times for the specific Ethernet link), we find the minimum required frame size is \( 2 \times d/(1.8 \times 10^8) \times BW - 1 \) (bits). Or approximately we choose \( 2 \times d/(1.8 \times 10^8) \times BW \) (bits). If \( d=2 \text{km} \), then minimum required frame size is: 222 bits.

Problem B
a) \( \frac{800 \text{m}}{2 \times 10^8 \text{m/sec}} + 4 \times \frac{20 \text{bits}}{100 \times 10^6 \text{bps}} = 4.8 \mu \text{sec} \)

b) First note, the transmission time of a single frame is give by \( \frac{1500}{100 \text{Mbps}} = 15 \text{ micro sec} \), longer than the propagation delay of a bit.
   - At time \( t = 0 \), both A and B transmit
   - At time \( t = 4.8 \mu \text{sec} \), both A and B detect a collision, and then abort.
   - At time \( t = 9.6 \mu \text{sec} \), last bit of B’s aborted transmission arrives at A.
   - At time \( t = 14.4 \mu \text{sec} \), first bit of A’s retransmission arrives at B.
   - At time \( t = 14.4 \mu \text{sec} + 1500 \text{bits}/(100 \times 10^6 \text{bps}) = 29.4 \mu \text{sec} \), A’s packet is completely delivered at B.

c) The line is divided into 5 segments by the switches, so the propagation delay between switches or between a switch and a host is given by \( \frac{800 \text{m}}{2 \times 10^8 \text{m/sec}} = 0.8 \mu \text{sec} \).

The delay from Host A to the first switch is give by 15 microsec (transmission delay), longer than propagation delay. Thus, the first switch will wait 16 = 15 + 0.8 + 0.2 (note, 0.2 is processing delay) till it is ready to send the frame to the second switch. Note that the store-and-forward delay at a switch is 15 microsec. Similarly each of the other 3 switches will wait for 16 microsec before ready for transmitting the frame. The total delay is: 16 × 4 + 15 + 0.8 = 79.8 micro sec.