CS331: Introduction to Artificial Intelligence
Written Assignment #3

Date handed out: May 13, 2015
Date due: May 20, 2015 at the start of class
Total: 24 points

This assignment is to be done individually.

1. Using the full joint probability distribution below, write out what the following probability distributions look like. Notice that the P is in boldface to emphasize that these are distributions (ie. probability tables). This means you have to write out the probability distributions for all uninstantiated random variables eg. for (a), write out P(Toothache=true) and P(Toothache=false).

<table>
<thead>
<tr>
<th>Toothache</th>
<th>Cavity</th>
<th>Catch</th>
<th>P(Toothache, Cavity, Catch)</th>
</tr>
</thead>
<tbody>
<tr>
<td>false</td>
<td>false</td>
<td>false</td>
<td>0.576</td>
</tr>
<tr>
<td>false</td>
<td>false</td>
<td>true</td>
<td>0.144</td>
</tr>
<tr>
<td>false</td>
<td>true</td>
<td>false</td>
<td>0.008</td>
</tr>
<tr>
<td>false</td>
<td>true</td>
<td>true</td>
<td>0.072</td>
</tr>
<tr>
<td>true</td>
<td>false</td>
<td>false</td>
<td>0.064</td>
</tr>
<tr>
<td>true</td>
<td>false</td>
<td>true</td>
<td>0.016</td>
</tr>
<tr>
<td>true</td>
<td>true</td>
<td>false</td>
<td>0.012</td>
</tr>
<tr>
<td>true</td>
<td>true</td>
<td>true</td>
<td>0.108</td>
</tr>
</tbody>
</table>

a) P(Toothache) (Note that this is a distribution, not a single number) [2 points]
b) P(Cavity) (Note that this is a distribution, not a single number) [2 points]
c) P(Toothache | Cavity) (Note that this is a distribution, not a single number) [4 points]

2. Show that the three forms of independence below are equivalent: [3 points]

\[
P(X,Y) = P(X)P(Y)
\]
\[
P(X|Y) = P(X)
\]
\[
P(Y|X) = P(Y)
\]
3. After your yearly checkup, the doctor has bad news and good news. The bad news is that you tested positive for a serious disease and that the test is 99% accurate (i.e., the probability of testing positive when you do have the disease is 0.99, as is the probability of testing negative when you don’t have the disease). The good news is that this is a rare disease, striking only 1 in 10,000 people of your age. Why is it good news that the disease is rare? What are the chances that you actually have the disease? [5 points]

4. (Adapted from Pearl 1988) Suppose you are witness to a nighttime hit-and-run accident involving a taxi in Athens. All taxis in Athens are blue or green. You swear, under oath, that the taxi was blue. Extensive testing shows that, under the dim lighting conditions, discrimination between blue and green is 75% reliable.
   a) Is it possible to calculate the most likely color for the taxi? (Hint: distinguish carefully between the proposition that the taxi *is* blue and the proposition that it *appears* blue.) [4 points]
   b) What if you know that 9 out of 10 Athenian taxis are green? What is the probability of the most likely color for the taxi? [4 points]