1. A school identification number includes 9 digits selected uniformly random in the set of 0, 1, 2, …, 9 with replacement. Also, the first digit cannot be 0. What is the probability that an identification number does not include either 0 or 3?

Solution: The total number of outcomes = \((9)(10)^8\)

Thus, 

\[ P = \frac{(8)(8)^8}{(9)(10)^8} = \frac{8^9}{(9)(10)^8} = 0.149 \]

2. If we place 3 dogs, 2 cats, and 9 rabbits into a private kennel at random in the veterinary clinic, what is the probability that all the dogs are together (in adjacent kennels)? Assume 1 animal per kennel and 14 available kennels.

Solution: If we simply put the 3 dogs in all 14 locations randomly without attention to neighbors there would be \(\binom{14}{3}\) cases. However, to keep the dogs together, consider the collection of 3 dogs as one, yielding now 12 different outcomes:

\[
\begin{align*}
&\{1\text{-Dog, 2-Dog, 3-Dog, 4-*,..., 14-*}\} \\
&\{1\text{-*, 2-Dog, 3-Dog, 4-Dog,..., 14-*}\} \\
&\cdots \\
&\{1\text{-*, 2\text{-*, ...}, 12-Dog, 13-Dog, 14-Dog}\}
\end{align*}
\]

If we instead randomly put the 3 dogs in all 14 locations there will be \(\binom{14}{3}\) cases. Thus, to find the probability that all dogs are placed together (adjacent kennels),

\[ P = \frac{3}{91} = 0.033 \]

3. Suppose we draw a sample of ten phones at random from a manufacturing line with a known defect rate of 2 percent. Find the probability that more than one of the phones in our sample is defective.

Solution: Here, we have a Binomial probability law \(b(k; N, p)\) with \(N = 10\) and \(p = P[\text{defect}] = 0.02\). So the probability of more than one defect in the sample is given as

\[
P[\text{more than 1 defect}] = \sum_{k=2}^{10} b(k; 10, 0.02) = 1 - \sum_{k=0}^{1} b(k; 10, 0.02)
\]

\[= 1 - (0.98)^{10} - \binom{10}{1} (0.02)(0.98)^9 \]

\[= 1 - (0.98)^{10} - (0.2)(0.98)^9 = 0.0162 \]

4. A fair coin is tossed 12 times. What is the probability of observing at least 4 heads?

Solution: The total number of outcomes = \(2^{12}\). We can compute the probability for choosing exactly 3, 2, 1, and 0 heads experiments and then use the rule of total probability.

To see 3 heads, there will be \(\binom{12}{3}\) cases yielding \(P_3 = \frac{\binom{12}{3}}{2^{12}} = \frac{220}{2^{12}}\).
*Could also use Bernoulli trials \( P_3 = \binom{12}{3} \left( \frac{1}{2} \right)^3 \left( \frac{1}{2} \right)^9 = \frac{\binom{12}{3}}{2^{12}} \). Two methods, same result.

To see 2 heads, there will be \( \binom{12}{2} \) cases yielding \( P_2 = \frac{\binom{12}{2}}{2^{12}} = \frac{66}{2^{12}} \).

To see 1 heads, there will be \( \binom{12}{1} \) cases yielding \( P_1 = \frac{\binom{12}{1}}{2^{12}} = \frac{12}{2^{12}} \).

To see 0 heads, there will be \( \binom{12}{0} \) cases yielding \( P_0 = \frac{\binom{12}{0}}{2^{12}} = \frac{1}{2^{12}} \).

Thus, \( P = 1 - P_0 - P_1 - P_2 - P_3 = 1 - \frac{1}{2^{12}} - \frac{12}{2^{12}} - \frac{66}{2^{12}} - \frac{220}{2^{12}} = \frac{2^{12} - 299}{2^{12}} = 0.927 \).

5. A technician needs to inspect 14 televisions one by one for defects. Suppose there are 5 defective and 9 non-defective items on the shelf. The technician will select at random and without replacement. (a) What is the probability that the first 4 items inspected are defective? and (b) What is the probability that from the first inspected 8 items there are 4 defective and 4 non-defective items?

**Solution:** Consider the order of inspection is a series of length 14. Thus, the total number of outcomes with 5 defective and 9 non-defective is \( \binom{14}{5} \).

(a) If the first 4 are fixed defective, the total number of outcomes will be \( \binom{14-4}{5-4} = \binom{10}{1} = 10 \).

Thus, \( P = \frac{10}{\binom{14}{5}} \).

(b) The total number of outcomes when the first 8 inspected include 4 defective and 4 non-defective is \( \binom{8}{4} \binom{14-8}{5-4} = \binom{8}{4} \binom{6}{1} = 420 \).

Thus, \( P = \frac{420}{\binom{14}{5}} = 0.21 \).