FINAL EXAMINATION

ECE 627

June 9, 2009, 2-3:50 pm

KEAR 305

Open book, open notes

1. An M+1 level DAC contains M unit current sources. The $i^{th}$ source has a value $I + dl_i$, where $I$ is the ideal value, and $dl_i$ is the error.

   a. Find the expression for the end-point matched INL$_n$ in terms of the $dl_i$ when $n$ sources are being used.

   b. Find the mean-square value (MSV) of INL$_n$ when the $dl_i$ are uncorrelated, and the MSVs of all errors are equal to $dl_i^2$. For what value of $n$ is the mean-square conversion error the largest? How large is it?

2. Find the NTF and STF of the ADC shown below.

3. The input of the delta-sigma ADC shown below satisfies $-1 \text{ V} < u(n) < 1 \text{ V}$; the LSB voltage of the quantizer is 0.4 V. What is the output voltage range of the first integrator?
Solutions.

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a. Actual output:

$$ A(n) = \sum_{i=1}^{n} (I + dl_i) = nI + \sum_{i=1}^{n} dl_i $$

End-point ideal output:

$$ A_e(n) = \frac{n}{M} \sum_{i=1}^{M} (I + dl_i) = nI + \frac{n}{M} \sum_{i=1}^{M} dl_i $$

$$ \text{INL}_n = \sum_{i=1}^{n} dl_i - \frac{n}{M} \sum_{i=1}^{M} dl_i = (1 - \frac{n}{M}) \sum_{i=1}^{n} dl_i $$

b. $\text{INL}_n^2 = \left[ n(1 - \frac{n}{M}) + (M - n) \frac{n^2}{M^2} \right] dl_i^2$

$$ = \frac{M - n}{M^2} \left[ n(M - n) + n^2 \right] dl_i^2 = \frac{n(M - n)}{M} dl_i^2 $$

$$ \frac{\partial \text{INL}_n^2}{\partial n} = 0 \rightarrow n_{\text{max}} = M/2 \quad (\text{or } M/2 \pm 1/2) $$

$$ \text{INL}_{n_{\text{max}}}^2 = M, dl_i^2/4, \quad \text{Relative MS error } e^2 = \frac{dl_i^2}{I^2} $$

FS output power $M I^2$. Relative MS error

$$ \frac{\text{INL}_n^2}{A_{FS}^2} = \frac{(M/4) e^2 I^2}{M^2 I^2} = \frac{e^2}{4M} \rightarrow 10 \log_{10} \left( \frac{e^2}{4M} \right) \text{ dB} $$
\[ Y = u + 2H(u-v) + \frac{H^2}{1-z^{-1}}(u-v) - z^{-1}E = V - E \]

\[ V = E + (H+1)^2U - [(H+1)^2 - 1]V - z^{-1}E \]

\[ (H+1)^2V = (H+1)^2U + (1-z^{-1})E \]

\[ (H+1)^2 = (1-z^{-1})^2 \]

\[ V = U + (1-z^{-1})^3E \]

\[ NTF = (1-z^{-1})^3 \]

\[ STF = 1 \]
\[ V = E + U + 3H(u-v) + 3H^2(u-v) + H^3(u-v) \]

\[ V = \frac{E}{(H+1)^3} + U = U + (1-z^{-1})^3E \]

\[ H(u-v) = z^{-1}(1-z^{-1})^2E \rightarrow -e(n-1) + 2e(n-2) - e(n-3) \]

\[ 1e_1(n) \leq 41e_1_{\text{max}} = 4V_{\text{LSB}}/2 = 0.8V \]