In the SAR ADC shown on the next page, initially $S_1$ and $S_{21}$ are closed, as well as $S_3$, $S_5$, $S_7$, $\ldots$, $S_{19}$. Then $S_1$ and $S_{21}$ open, and $S_2$ closes to sample $V_{in}$ at node A. Afterwards, the switches $S_3 - S_{20}$ are used to generate the comparison voltages $V_A$ at node A to find the bits $b_i$ of the output word. The element values are $C = 0.4$ pF, $C_s = 0.1$ pF and $R = 1$ kΩ. $V_{ref} = 3$ V.

a. What is the resolution of the ADC?

b. What are the MSB and LSB voltage steps in $V_A$?

c. Find the expression for $V_A$ in terms of $V_{in}$, $b_i$, $C$ and $V_{ref}$.

d. What will be the value of the voltage $V_A$ after the first 3 MSB cycles if $V_{in} = 0.8V_{ref}$?

e. What is the worst-case error in $V_A$ if the resistors have a 1% matching error?

f. What is the worst-case DNL error if the capacitors have a 0.2% matching error?
Solutions

(a) By inspection $N = 9$

(b) MSB step $(S_3 - S_4)$:

\[
\begin{align*}
S_4 \to & V_A \\
S_3 \downarrow & C_T - C \\
V_\text{ref} \downarrow & \frac{C}{C_T}
\end{align*}
\]

\[
C_T = 2C + C_S + 3C \left(1 + 0.5 + 0.25\right) = 7.25C + C_S = 3 \text{ pF}
\]

\[
\Delta V_{A_{\text{MSB}}} = \frac{C}{C_T} V_{\text{ref}} = 0.4 \text{ V}
\]

\[
\Delta V_{A_{\text{LSB}}} = \frac{C/4}{C_T} \frac{V_{\text{ref}}}{64} = 1.5625 \text{ mV}
\]

\[
\frac{\Delta V_{A_{\text{MSB}}}}{\Delta V_{A_{\text{LSB}}}} = 256 = 2^8
\]

(c) \( V_A = \left(V_{\text{ref}} C/C_T\right) \left(b_1 + b_2 2^{-1} + \ldots + b_9 2^{-8}\right) \)

For \( V_{\text{in}} = 0 \):

\[
V_A = 0.4 \sum_{i=1}^{9} b_i 2^{-i+1} = 0.8 \sum_{i=1}^{9} b_i 2^{-i} \text{ V}
\]

\[
V_A = V_A - \frac{2C}{C_T} V_{\text{in}} = 0.8 \sum_{i=1}^{9} b_i 2^{-i} - 0.266 V_{\text{in}}
\]

(d) \( V_A = 0.8 \left(0.5 + 0.25\right) - 0.64 = -40 \text{ mV} \)
(2) Worst case: \( R \) 's below \( V_{ref}/8 \) too small, above too large (or vice versa). Then, the string current is \( i = \frac{V_{ref}}{(56.56 + 7.92)} \)

\[ i \approx 46.526 \text{ mA} \]

\[ V_8 = \frac{V_{ref}}{8} = 0.375 \implies 0.3685 \text{ V} \]

\[ V_{64} = \frac{V_{ref}}{64} = 0.0469 \implies 0.04606 \text{ V} \]

In the worst case, all caps at \( V_{ref}/8 \) and \( V_{ref}/64 \) are outside, so the error in \( V_A \) is

\[ \Delta V_A = \frac{C}{C_T} (1 + 0.5 + 0.25) (\Delta V_8 + \Delta V_{64}) = \]

\[ \Delta V_A = \frac{0.4}{3} 1.75 (6.5 + 0.24) \text{ mV} \approx 1.713 \text{ mV} \]

(3) Worst case: 100...0 \( \rightarrow \) 011...1.

Assume MSB cap \( C_{MSB} = 11.002 \text{ pF} \), and all others \( 0.998C_{non} \). Then \( C_T = 0.998(6.25C + C_3) + 1.022C = 2.9955 \text{ pF} \), Change in \( V_A \) value:

\[ \Delta V_A = V_{ref} \left[ \frac{1.022C}{C_T} - 0.998 \left( 2^{-1} + \ldots + 2^{-8} \right) \frac{C}{C_T} \right] \]
\[ \Delta V_A = V_{ih} \frac{C}{C_T} \left[ 1.002 - 0.992 \times (1 - 2^{-9}) \right] \]

\[ \approx 0.4006 \times 0.00777 \approx 3.1 \mu V \sim 2 \text{ LSBs} \]